EXPONENTS

OUTCOMES:

1. Simplify expressions and solve equations using the laws of exponents for rational exponents where

LAWS OF EXPONENTS (BASICS)

- When the same bases are multiplied the exponents are added

$$\underline{a^n \times a^m = a^{n+m}}$$

- When the same bases are divided the exponents are subtracted

$$\underline{a^n \div a^m = a^{n-m}}$$

- When a power is raised to an exponent, the exponents are multiplied

$$\underline{(a^n)^m = a^{n \times m}}$$

- When a power is under a root sign the exponent is divided by the root

$$\sqrt[m]{a^n} = a^{\frac{n}{m}}$$

- Anything to the power of zero is one

$$\underline{a^0 = 1}$$

- When a power has a negative exponent, the power is inverted (placed under 1) and the exponent becomes positive

$$a^{-n} = \frac{1}{a^n}$$

REVISION OF GRADE 8 and 9 EXPONENTS BASICS

Simplify the following expressions without the use of a calculator

Examples:

a)	$2^5 . 2^3 = 2^{5+3} = 2^8$
b)	$x^3 . x = x^{3+1} = x^4$
c)	$\frac{3^7}{3^2} = 3^{7-2} = 3^5$
d)	$\frac{2^{14}x^{10}y^{20}}{2^7x^4y^5} = 2^7x^6y^{15}$
e)	$(2^5)^3 = 2^{5\times3} = 2^{15}$
f)	$3x(3^3x^4)^4 = 3x(3^{12}x^{16}) = 3^{1+12}.x^{1+16} = 3^{13}x^{17}$
g)	$a^0 = 1$
h)	$2x^0 = 2(1) = 2$
i)	$2^{-3} = \frac{1}{2^3}$
j)	$(3x^4)^{-2} = 3^{1\times-2}.x^{4\times-2} = 3^{-2}x^{-8} = \frac{1}{3^2x^8}$
k)	$\left(\frac{3x}{5}\right)^{-2}$ $= \left(\frac{5}{3x}\right)^2$(fractions swop from top to bottom) $= \frac{5^2}{3^2x^2}$

REVISION OF GRADE 10 EXPONENTS

1	**Simplify the following expressions without the use of a calculator**		
1.1	$x^2y^5 \times x^4y$	**1.2**	$6a^0 + (6a)^0$
1.3	$(-3x^2y^3)^2$	**1.4**	$\dfrac{2x^2y.4x^2y^3}{4x^7y^6}$
1.5	$15x^4y^6 \times 2y^2x^0$	**1.6**	$\dfrac{3x^3}{27x^9} \times (3x^3)^3$
1.7	$\dfrac{2(x^2)^4(x^4)^2(2x^3)^2}{16(x^4)^5}$	**1.8**	$\dfrac{(3xy^{-2})^{-2}}{3x^{-2}y}$
1.9	$\left(\dfrac{12y^4}{4y^6}\right)^{-2}$	**1.10**	$\dfrac{6(x^2y)^3(x^{-2}y)^{-1}}{3x^{-3}y^5}$

MULTIPLICATION AND DIVISION OF POWERS WITH NUMERICAL BASES

2	**Simplify the following expressions without the use of a calculator**

WORKED EXAMPLE 1	**WORKED EXAMPLE 2**
$\dfrac{2^{x+2}.4^{x+1}}{8^{x-1}}$	$\dfrac{8^n.6^{n-3}.9^{1-n}}{16^{n-1}.3^{-n}}$
$=\dfrac{2^x.2^2.(2^2)^{x+1}}{(2^3)^{x-1}}$	$=\dfrac{(2^3)^n.(2^1\times 3^1)^{n-3}.(3^2)^{1-n}}{(2^4)^{n-1}.3^{-n}}$
$=\dfrac{2^x.2^2.2^{2x}.2^2}{2^{3x}.2^{-3}}$	$=\dfrac{2^{3n}.2^n.2^{-3}.3^n.3^{-3}.3^2.3^{-2n}}{2^{4n}.2^{-4}.3^{-n}}$
$=\dfrac{2^{3x}.2^4}{2^{3x}.2^{-3}}$	$=\dfrac{2^{4n-3}.3^{-n-1}}{2^{4n-4}.3^{-n}}$

$=\dfrac{2^{3x+4}}{2^{3x-3}}$ $=2^{3x+4-(3x-3)}$ $=2^{3x+4-3x+3}$ $=2^{7}$	$=2^{4n-3-(4n-4)}.3^{-n-1-(-n)}$ $=2^{4n-3-4n+4}.3^{-n-1+n}$ $=2.3^{-1}=2\times\dfrac{1}{3}$ $=\dfrac{2}{3}$

2.1	$\dfrac{27^{x}.9^{x+1}}{3^{5x+3}}$	**2.2**	$\dfrac{18^{x}.24^{x+1}}{12^{3x}.4^{-x}}$
2.3	$\dfrac{12^{x-2}.2^{x+2}}{8^{x}.3^{x-4}}$	**2.4**	$\dfrac{9^{x-2}.2^{3x+2}}{8^{x}.3^{2x-1}}$
2.5	$\dfrac{4^{n+2}.9^{n-1}}{72^{n}.2^{1-n}}$	**2.6**	$\dfrac{10^{x}.25^{x+1}}{5^{x}.50^{x-1}}$
2.7	$\dfrac{6^{n+2}\times 10^{n-2}}{4^{n}\times 15^{n-2}}$	**2.8**	$\dfrac{6^{x}.9^{x+1}.2}{27^{x+1}.2^{x-1}}$
2.9	$\dfrac{54^{x+1}.36^{x-1}}{24^{2x-3}}$	**2.10**	$\dfrac{27^{x+1}.18^{x-1}}{36^{3-x}}$

ADDITION AND SUBTRATION OF POWERS WITH NUMERICAL BASES

3	**Simplify the expressions without the use of a calculator**

WORKED EXAMPLE 1 $\dfrac{\mathbf{2^{n+1}-2^{n-1}}}{\mathbf{2^{n}}}$ $=\dfrac{2^{n}.2^{1}-2^{n}.2^{-1}}{2^{n}}$ $=\dfrac{\cancel{2^{n}}\left(2-\frac{1}{2}\right)}{\cancel{2^{n}}}$	**WORKED EXAMPLE 2** $\dfrac{\mathbf{5^{a-2}.2^{a+2}}}{\mathbf{10^{a}-10^{a-1}.2}}$ $=\dfrac{5^{a}.5^{-2}.2^{a}.2^{2}}{(2\times5)^{a}-(2\times5)^{a-1}.2}$ $=\dfrac{5^{a}.5^{-2}.2^{a}.2^{2}}{2^{a}.5^{a}-2^{a}.2^{-1}.5^{a}.5^{-1}.2}$

$=\frac{3}{2}$	$=\frac{5^{a}.5^{-2}.2^{a}.2^{2}}{2^{a}.5^{a}-2^{a-1+1}.5^{a-1}}$ $=\frac{5^{a}.5^{-2}.2^{a}.2^{2}}{2^{a}.5^{a}-2^{a}.5^{a}.5^{-1}}$ $=\frac{5^{a}.5^{-2}.2^{a}.2^{2}}{2^{a}.5^{a}(1-5^{-1})}$ $=\frac{\frac{4}{5^{2}}}{\frac{4}{5}}=\frac{4.5}{5^{2}.4}=5^{-1}=\frac{1}{5}$

3.1	$2^{x+1}+3.2^{x}$	**3.2**	$9^{x-1}+3^{2x}$
3.3	$5.2^{x}-2^{x+2}$	**3.4**	$9^{x}+3^{2x+1}$
3.5	$2^{2x-1}+4^{x+1}$	**3.6**	$\frac{2^{x+2}-2^{x}}{3.2^{x}}$
3.7	$\frac{6a^{4}+9a^{4}}{5a^{2}}$	**3.8**	$\frac{3^{x}+3^{x+1}}{8}$
3.9	$\frac{8^{2x+1}+4^{3x-1}}{2^{3x}}$	**3.10**	$\frac{2^{x+2}-2^{x+3}}{2^{x+1}-2^{x+2}}$
3.11	$\frac{4^{x}+2^{2x-1}}{2^{2x-1}}$	**3.12**	$\frac{2.3^{x+2}+3^{x-3}}{5.3^{x-2}}$
3.13	$\frac{3.2^{x}-2^{x-1}}{2^{x}+2^{x+2}}$	**3.14**	$\frac{2^{2x}-2^{x}}{2^{x}-1}$
3.15	$\frac{9^{x}-9}{3^{x}-3}$		
4	**Solve the following exponential equations.** **Exponential equations have the unknown or variable in the exponent.**		

GRADE 11 – WORKSHEET 1

WORKED EXAMPLE 1

Solve for $\boldsymbol{x}$: $3^x = 9^{x+1}$

$$3^x = (3^2)^{x+1}$$
$$3^x = 3^{2x+2}$$
$$x = 2x + 2$$
$$x - 2x = 2$$
$$-x = 2$$
$$x = -2$$

WORKED EXAMPLE 2

Solve for $\boldsymbol{x}$: $3^x = 9^{x+1}$

$$3^{2x+1} - 3^{2x} = 18$$
$$3^{2x}.3^1 - 3^{2x} = 18$$
$$3^{2x}(3-1) = 18$$
$$3^{2x}.2 = 18$$
$$3^{2x} = 9$$
$$3^{2x} = 3^2$$
$$2x = 2$$
$$x = 1$$

4.1	$2^x = 4^{x+1}$	**4.2**	$8^{x+1} = 32$
4.3	$7^{3x+1} = 49^{x-1}$	**4.4**	$5.9^{x+1} = 15$
4.5	$2.3^{2x+1} = 54$		

The following equations have more than one term on each side

- Rearrange the equations so the unknown exponents are on one side.
- Factorise by taking out a common factor.
- Solve the remaining exponential equation.

WORKED EXAMPLE 3	**WORKED EXAMPLE 4**
$3^x - 3^{x-2} = 24$ $3^x - 3^x.3^{-2} = 24$ $3^x(1-3^{-2}) = 24$ $3^x\left(\frac{8}{9}\right) = 24$ $3^x = 24 \times \frac{9}{8}$ $3^x = 27$ $3^x = 3^3$ $x = 3$	$2^{x+1} + 2^3 2^{-x} = 17$ $2^x.2 + 2^3\left(\frac{1}{2^x}\right) = 17$ LET:$2^x = k$ $k.2 + 2^3\left(\frac{1}{k}\right) = 17$ $2k^2 + 8 = 17k$ $2k^2 - 17k + 8 = 0$ $(2k-1)(k-8) = 0$ $k = \frac{1}{2}\ or\ k = 8$ $k = 2^{-1}\ or\ k = 2^3$ $2^x = 2^{-1}\ or\ 2^x = 2^3$ $x = -1\ or\ x = 3$

4.6	$2^x + 2^{x+1} = 24$	**4.7**	$5^{3x} - 5^{3x-1} = 4$
4.8	$2^{x-1} + 2^{2+x} = 144$	**4.9**	$5^{x+2} + 5^x = 26$
4.10	$3^{x+1} = 8 + 3^{x-1}$	**4.11**	$3^{x+1} = 3^x + \frac{2}{3}$
4.12	$2^{3x+1} + 2^{3x} = 12$	**4.13**	$3^{x+2} - 3^{x+2} = 486$
4.14	$5^{2x+4} - 25^{x-1} = 78\,120$	**4.15**	$3^{2-x} - 3^{-x-3} = \frac{242}{9}$
4.16	$4^{x+1}.3 + 5.2^{2x-1} - 7 = \frac{1}{4}$		