

ARITHMETIC SEQUENCES AND SERIES

OUTCOMES:

1. Recognise and arithmetic sequences.
2. Find the general arithmetic sequences.
3. Answer questions based on the arithmetic sequence, like finding the position in a sequence.
4. Find the sum of an arithmetic sequence.

INTRODUCTION:

In the previous grades you were introduced to numeric number patterns which is a **sequence** of numbers that follow a specific pattern. An example of a **linear pattern (arithmetic sequence)** is one where there is a **constant difference** between consecutive terms. In other words, the same number will be added to, or subtracted from each consecutive term.

A **sequence** is an ordered list of numbers or objects

An arithmetic sequence: 5; 9; 13; 17; 21 can be presented as follows:

- $T_1 = a$ = the first term
- T_3 = the third term: 3 is the **position** of the term in the sequence
- 21 is the **value** of the fifth term: $T_5 = 21$
- 4 is the constant/common difference (d) between two consecutive terms: $T_5 - T_4 = \text{constant difference}$
- n = number of terms

THE GENERAL TERM T_n is given by:

$$T_n = a + (n - 1)d$$

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WORKED EXAMPLE 1:

Given the sequence: 2; 5; 8;

a)	Determine the general term of the sequence $a = 2$ $d = 5 - 2 = 3$ $T_n = a + (n - 1)d$ $T_n = 2 + (n - 1)3$ $T_n = 2 + 3n - 3$ $T_n = 3n - 1$
b)	Use the general rule to determine the 40 th term. $T_{40} = 3(40) - 1$ $T_{40} = 119$
c)	Which term in the pattern will be equal to 2012? $T_n = 3n - 1$ $2012 = 3n - 1$ $2013 = 3n$ $n = 671$

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WORKED EXAMPLE 2:

Find the number of terms in the arithmetic sequence:

$$-2; -6; -10; \dots - 150$$

$$a = -2$$

$$d = -6 - (-2) = -4$$

$$T_n = -150$$

$$T_n = a + (n - 1)d$$

$$-150 = -2 + (n - 1)(-4)$$

$$-150 = -2 - 4n + 4$$

$$-150 = 2 - 4n$$

$$-152 = -4n$$

$$n = \frac{-152}{-4}$$

$$n = 38$$

WORKED EXAMPLE 3:

Determine the first three terms of an arithmetic sequence if the constant difference is 10 and the fourth term is 30.

$$d = 10$$

$$T_4 = 30$$

$$T_n = a + (n - 1)d$$

$$30 = a + (4 - 1)(10)$$

$$a = 0$$

$$0; 10; 20; 30$$

WORKED EXAMPLE 4:

In an arithmetic sequence the 2nd term is 9 and the 5th term is 21. Determine:

- | | |
|----|--|
| a) | The first three terms of the sequence. |
|----|--|

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	$T_2 = 9$ $T_5 = 21$ $T_n = a + (n - 1)d$ $9 = a + (2 - 1)d$ $9 = a + d$ equation 1 $T_n = a + (n - 1)d$ $21 = a + (5 - 1)d$ $21 = a + 4d$ equation 2 $a = 9 - d$ use equation 1 to make a subject of the formula $21 = a + 4d$ $21 = 9 - d + 4d$ $d = 4$ $a = 9 - d$ $a = 9 - 4$ $a = 5$ 5; 9; 13
b)	The 60 th term. $T_n = a + (n - 1)d$ $T_{60} = 5 + (60 - 1)(4)$ $T_{60} = 241$
WORKED EXAMPLE 5: 2p – 3; 3p – 1; 5p – 2 are the first three terms of an arithmetic sequence.	
a)	Determine the value of p .

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	$d = T_2 - T_1$ $d = 3p - 1 - (2p - 3)$ $d = 3p - 1 - 2p + 3$ $d = p + 2$	$d = T_3 - T_2$ $d = 5p - 2 - (3p - 1)$ $d = 5p - 2 - 3p + 1$ $d = 2p - 1$
	$T_2 - T_1 = T_3 - T_2$ $p + 2 = 2p - 1$ $p - 2p = -1 - 2$ $-p = -3$ (divide -1) $p = 3$	
b)	The first three terms of the sequence.	
	$T_1 = 2p - 3$ $T_1 = 2(3) - 3$ $T_1 = 6 - 3$ $T_1 = 3$	$T_2 = 3p - 1$ $T_2 = 3(3) - 1$ $T_2 = 9 - 1$ $T_2 = 8$
c)	Determine the term equal to 2013 $T_n = a + (n - 1)d$ $2013 = 3 + (n - 1)(5)$ $2013 = 3 + 5n - 5$ $n = 403$	

HOMEWORK ACTIVITY

1	Determine the 11 th term of the arithmetic sequence 4; 7; 10; ...
2	Which term of the arithmetic sequence $-2; 3; 8; 13 \dots$ is equal to 493?
3	Given the arithmetic sequence: 35; 28; 21; Calculate which

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	term of the sequence will have a value of -140 .
4	In an arithmetic sequence the 3^{th} term is -4 and the 7^{th} term is -20 .
a)	Determine the first four terms
b)	Determine the 30^{th} term.
5	The first three terms of an arithmetic sequence are $3p - 4$; $4p - 3$ and $7p - 6$.
a)	Determine the value of p .
b)	Determine the first three terms of the sequence.
c)	Determine the value of the 16^{th} term.
6	Given the following sequences: 3; 8; 13; 18; ...
a)	The general term
b)	The 20^{th} term
c)	Which term of the sequence is equal to 223
7	In an arithmetic sequence, $T_3 = -2$ and $T_8 = 23$. Determine the first term and the constant difference.
8	Find the number of terms in the arithmetic sequence: -5 ; -11 ; -17 ; ...; -491
9	The first three terms of an arithmetic sequence is: $x - 8$; x ; $2x - 5$. Determine:
a)	The value of x .
b)	The general term.

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c) The value of the 115^{th} term.

SERIES: A **series** is created by **adding the terms of a sequence**.

- 2; 5; 8; 11; 14 THIS IS AN ARITHMETIC SEQUENCE
- $2 + 5 + 8 + 11 + 14$ THIS IS AN ARITHMETIC SERIES

The sum of a sequence is labelled as S_n or the Greek symbol Σ

Hence the sum of the first n terms of an arithmetic series is given by the

formule:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Or

$$S_n = \frac{n}{2} [a + l] \quad l = \text{LAST TERM}$$

WORKED EXAMPLE 1:

How many terms of the arithmetic series: $1 + 4 + 7 + \dots$ will add up to 145.

$$a = 1; d = 3; n = ?; S_n = 145$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$145 = \frac{n}{2} [2(1) + (n - 1)3]$$

$$290 = n[2 + 3n - 3]$$

$$290 = n[3n - 1]$$

$$0 = 3n^2 - n - 290$$

$$0 = (3n + 29)(n - 10)$$

$$n = -\frac{29}{3} \text{ or } n = 10$$

$$\therefore n = 10; n \in \mathbb{N}$$

WORKED EXAMPLE 2:

Consider the arithmetic series $-4 - 1 + 2 + \dots$. Calculate the smallest

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value of n for which $S_n > 300$

$$= -4; d = 3; n = ?; \text{ let } S_n = 300$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$300 = \frac{n}{2} [2(-4) + (n-1)3]$$

$$600 = n[-8 + 3n - 3]$$

$$0 = 3n^2 - 11n - 600$$

$$\therefore n = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(3)(-600)}}{2(3)}$$

$$\therefore n = 16,09 \text{ or } n = -12,43$$

$\therefore$ The smallest possible value of n is 17

HOMEWORK ACTIVITY

1	How many terms in the arithmetic series $34 + 40 + 46 + \dots$ will add up to 1 530?
2	The sum of the first ten terms of an arithmetic sequence is 60. The sum of the 2^{nd} and 6^{th} terms is 9. Determine the series.
3	The common difference of an arithmetic sequence is 3 and the sum of its first 16 terms is -120 . Find its first term.
4	Determine the sum to 50 terms of the arithmetic series whose seventh term is 33 and whose fifteenth term is 65.
5	How many terms of the series $-3 + 5 + 13 + \dots$ must be added to give a sum of 261?
6	Determine $5 + 12 + 19 + \dots + 54$ ($S_n = 236$)
7	How many terms of the arithmetic series $3 + 7 + 11 + \dots$ will add up to 210? ($n = 10$)