SIGMA NOTATION

OUTCOMES:

Sigma notation

INTRODUCTION:

$S_n = \frac{n}{2}[2a + d(n-1)]$ - **ARITHMETIC SERIES**

$S_n = \frac{a(1-r^n)}{1-r}$ *or* $S_n = \frac{a(r^n-1)}{r-1}$ – **GEOMETRIC SERIES**

The Greek Letter sigma $\sum$

Sigma is the capital letter S in the Greek alphabet. It signifies that we are dealing with the sum of the indicated terms of a SERIES.

We end with p=6

$$\sum_{p=2}^{6}$$ **General term** (T_p)

We start with p=2

$$\sum_{k=1}^{n} T_k = T_1 + T_2 + T_3 + \ldots + T_n = S_n$$

$$n = \text{number of terms}$$
$$n = \text{top} - \text{bottom} + 1$$

GRADE 12 – WORKSHEET 3

1. Evaluate/expand the following:

a)

$$\sum_{t=0}^{90}(3t-1)$$

$T_1 = 3(0) - 1 = 0 - 1 = -1$

$T_2 = 3(1) - 1 = 3 - 1 = 2$

$T_3 = 3(2) - 1 = 6 - 1 = 5$

THE LAST TERM IS $T_{LAST} = 3(90) - 1 = 270 - 1 = 269$

$-1 + 2 + 5 \ldots + 269$

b)

$$\sum_{k=0}^{12}(7-12k)$$

$T_1 =$

$T_2 =$

$T_3 =$

THE LAST TERM IS $T_{LAST} =$

$7 - 5 - 17 \ldots . - 137$

c)

$$\sum_{r=2}^{5}2^r$$

$T_1 =$

$T_2 =$

$T_3 =$

THE LAST TERM IS $T_{LAST} =$

$4 + 8 + 16 + \ldots 32$

2. Determine n in each series:

a)

$$\sum_{r=1}^{n}\left(\frac{5r}{4}\right) = 150$$

$$T_1 = \frac{5(1)}{4} = \frac{5}{4}$$

$$T_2 = \frac{5(2)}{4} = \frac{10}{4} = \frac{5}{2}$$

$$T_3 = \frac{5(3)}{4} = \frac{15}{4}$$

$$a = \frac{5}{4}$$

$$d = \frac{5}{4}$$

$$S_n = \frac{n}{2}[2a + d(n-1)]$$

$$150 = \frac{n}{2}\left[\left(2 \times \frac{5}{4}\right) + \frac{5}{4}(n-1)\right]$$

$$150 = \frac{n}{2}\left[\frac{5}{2} + \frac{5}{4}n - \frac{5}{4}\right]$$

$$150 = \frac{n}{2}\left(\frac{5}{4} + \frac{5}{4}n\right)$$

$$150 = \frac{5n}{8} + \frac{5n^2}{8}$$

$$1200 = 5n + 5n^2$$

$$240 = n^2 + n$$

$$0 = n^2 + n - 240$$

$$0 = (n+16)(n-15)$$

n can only be a positive whole number, $\therefore n = 15$

b)

$$\sum_{r=1}^{n}(22-3r)=44$$

$T_1=19$ $T_2=16$ $T_3=13$	$a=19$ $d=-3$

$$44=\frac{n}{2}[2(19)-3(n-1)]$$

$$88=n[38-3n+3]$$

$$88=-3n^2+41n$$

$$0=-3n^2+41n-88$$

$$0=3n^2-41n+88$$

USE THE FORMULA

n can only be a positive whole number, $\therefore n=11$

c)

$$\sum_{k=1}^{n}3(\ 2)^{k-1}=381$$

3. Write the following series in sigma notation: $5+8+11+14+17$

First calculate the general term for the series where: $a=5$ and $d=3$

Hence:

$$T_n=a+(n-1)d$$
$$T_n=5+(n-1)3$$
$$T_n=5+3n-3$$
$$T_n=3n+2$$

Write the formula now in sigma notation:

$$\sum_{n=?}^{?}3n+2$$

Bottom value is the first term which is equal to 5. $3n + 2 = 5$ $n = 1$	Top value is the last term which is equal to 17. $3n + 2 = 17$ $n = 5$

$$\sum_{n=1}^{5} 3n + 2$$

4. Expand and then calculate

$$\sum_{r=3}^{15} 3(-2)^{r-1}$$

$T_1 = 3(-2)^{r-1} = 3(-2)^{3-1} = 3(-2)^2 = 12$

$T_2 = -24$

$T_3 = 48$

The last term is equal 49152

$a = 12$

$r = 2$

$\boldsymbol{n = 15 - 3 + 1 = 13}$

$S_n = ?$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{13} = \frac{12(2^{13} - 1)}{2 - 1}$$

$S_{13} = 98\,292$

HOMEWORK ACTIVITY

1	Determine the value of the following. $$\sum_{r=0}^{10} (2r + 5)$$

2	Write the following in sigma notation $7 + 10 + 13 + .. + 25$
3	Write $5 + 8 + 11 + 14 + 17 + 20 + 23 + 26 + 29$ in sigma notation.
4	Find the sum of the following:
a)	$\sum_{n=1}^{3} 128\left(\frac{1}{2}\right)^{n-1}$
b)	$\sum_{k=0}^{4} (-3)^k$
c)	$\sum_{a=2}^{50} (5 - 2a)$
5	Determine m in the following arithmetic series: $\sum_{i=0}^{m} (1 - 3i) = -671$
6	Given the arithmetic series: $-7 - 3 + \cdots + 173$
a)	How many terms are there in the series?
b)	Calculate the sum of the series.
c)	Write the series in sigma notation.
7	Given: $\sum_{k=1}^{n} 3(2)^{1-k} = 5{,}8125$. Calculate the value of n.