EUCLIDEAN GEOMETRY: PROPORTIONALITY

OUTCOMES:

1. Prove the theorem that states that a line drawn parallel to one side of a triangle, divide the other two sides proportionally.
2. Answer riders using the proportionality theorem.

INTRODUCTION:

We need to revise the concept of ratio and proportion in order to be successful with this section of the work.

We know that a ratio compares 2 quantities of the same unit.

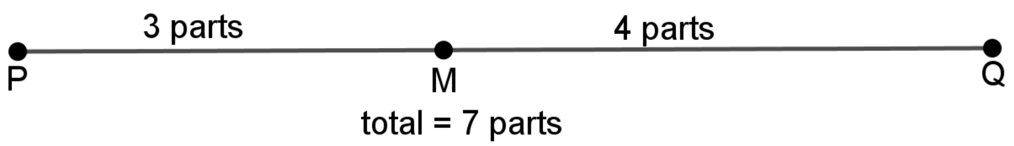
Example

$1:3$ means 1 part to 3 parts

Mixing Oros: To make a perfect drink we need 1 part of Oros and 3 parts of water. Remember that the unit must be the same.

WORKED EXAMPLE 1:

If M divides the line PQ in the ratio $3:4$ as shown below.



Note that the ratio is NOT the length of the line. We can use variables to indicate the equal parts of the ratio. Therefore $\mathrm{PM} = 3\mathrm{k}$ and $\mathrm{MQ} = 4\mathrm{k}$. Determine the following ratios:

a)	$\frac{PM}{PQ} = \frac{3k}{7k} = \frac{3}{7}$
b)	$\frac{PQ}{QM} = \frac{7k}{4k} = \frac{7}{4}$
c)	$\frac{QM}{MP} = \frac{}{} = \frac{}{}$

d)	$\frac{QP}{PM} = \frac{}{} = \frac{}{}$
e)	The lengths of PM if PQ $=$ 35 units?

$\frac{QP}{PM} = \frac{7}{3}$ $\frac{35}{PM} = \frac{7}{3}$ $\therefore 35 \times 3 = PM \times 7$ $\therefore PM = 15$ units	$7\text{k} = 35$ units $\text{k} = 5$ units $\text{PM} = 3(5) = 15$ units

A **PROPORTION** is an equation of equivalent ratio's.

Using a small measuring unit in the ratio $1:3$ might not fill the glass to mix a glass of Oros, but we know that $1:3 = 4:12$. Remember that the ratio's and proportion can be written in fractional form.

$$\therefore \frac{1}{3} = \frac{4}{12}$$

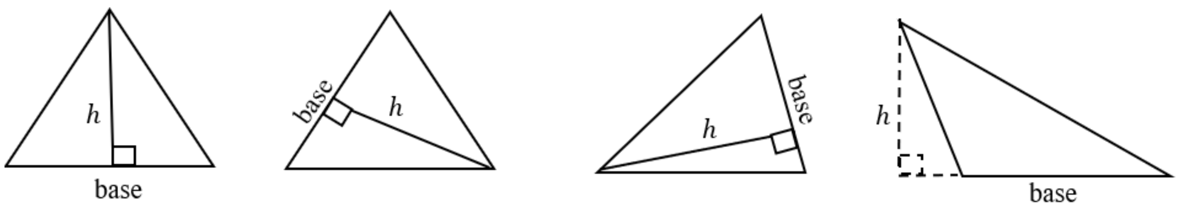
IF	$\frac{a}{b} = \frac{c}{d}$ $\frac{1}{3} = \frac{4}{12}$	THEN	$ad = bc$	$3 \times 4 = 1 \times 12$ Cross multiplication
			$\frac{b}{a} = \frac{d}{c}$	$\frac{3}{1} = \frac{12}{4}$ Inverted
			$\frac{a+b}{b} = \frac{c+d}{d}$	$\frac{1+3}{3} = \frac{4+12}{12}$ $\therefore \frac{4}{3} = \frac{16}{12}$
			$\frac{a}{c} = \frac{b}{d}$	$\frac{1}{4} = \frac{3}{12}$

			$\frac{d}{b} = \frac{c}{a}$	$\frac{12}{3} = \frac{4}{1}$

CONCEPTS AND SKILLS

Important information that we need before we can prove the first theorem of Area of Triangles.

$$\textbf{Area} = \frac{1}{2}\,\textbf{base} \times \perp \textbf{height}$$

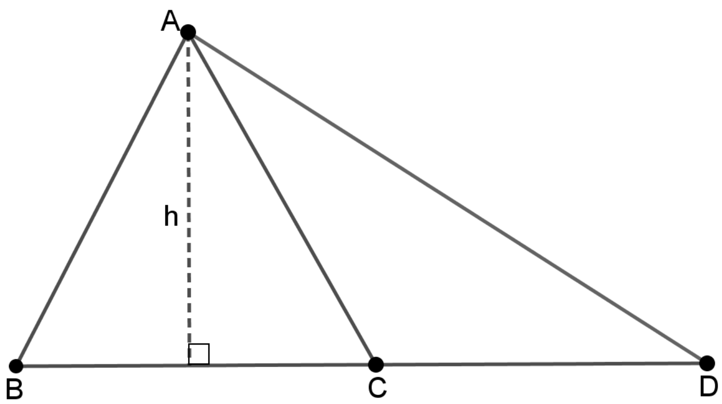


NOTE:

The perpendicular height is dropped onto the base.

Therefore:

Triangles which share a common vertex (a point where two points meet) have the same height.

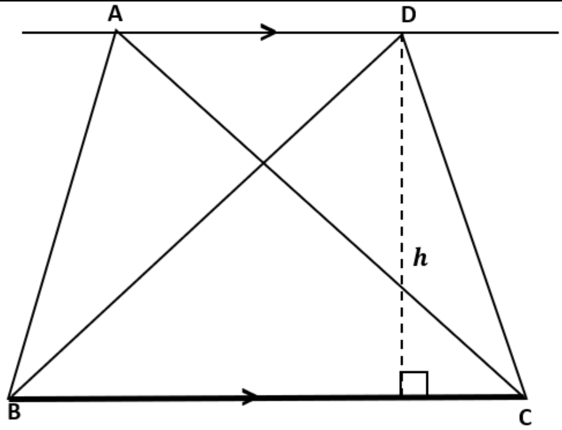


$$\text{Area } \Delta\, \text{ABC} = \frac{1}{2} \times \text{BC} \times \text{h}$$

$$\text{Area } \Delta\, \text{ACD} = \frac{1}{2} \times \text{CD} \times \text{h}$$

h is the same for BOTH triangles

COMMON VERTEX



$$\text{Area } \Delta\ ABC = \frac{1}{2}\ BC \times h$$

$$\text{Area } \Delta\ DBC = \frac{1}{2}\ BC \times h$$

Triangles on the same base and between the same parallel lines are equal in area.

THEOREM 1

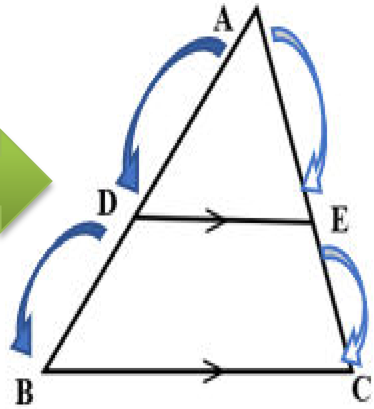
A line drawn parallel to one side of a triangle divides the other two sides proportionally.

[prop theorem; $\boldsymbol{DE//BC}$]

Given: ΔABC with DE // BC.

Required to prove:

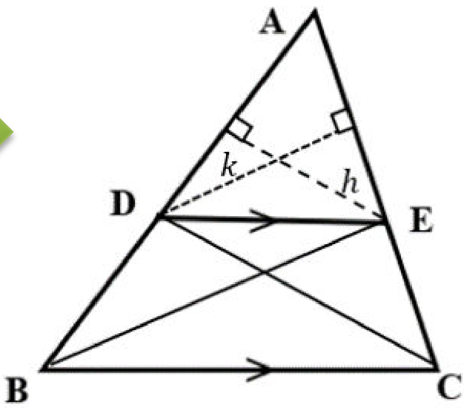
$$\frac{AD}{DB} = \frac{AE}{EC}$$



CONSTRUCTION:

Join BE and DC.

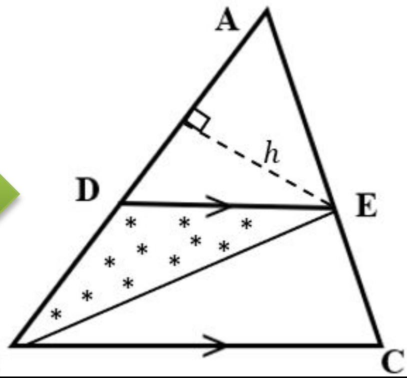
Draw height h relative to base AD and height k relative to base AE



PROOF:

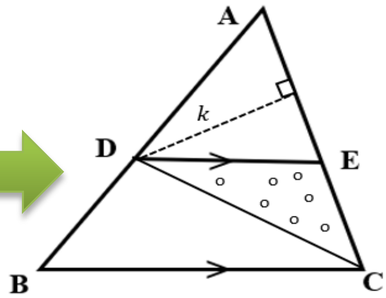
$$\frac{\text{Area } \Delta ADE}{\text{Area } \Delta BDE} = \frac{\frac{1}{2} AD \times h}{\frac{1}{2} DB\ \times h} = \frac{\text{AD}}{\text{DB}}$$

COMMON VERTEX E



$$\frac{\text{Area } \Delta \text{ADE}}{\text{Area } \Delta \text{CED}} = \frac{\frac{1}{2} AE \times k}{\frac{1}{2} EC \ \times k} = \frac{\text{AE}}{\text{EC}}$$

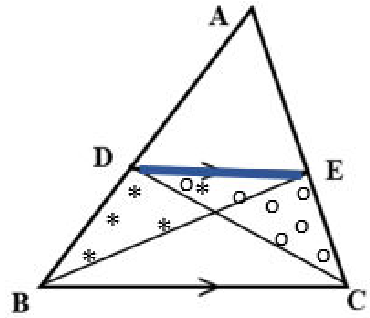
COMMON VERTEX D



Area of ADE is common
Area Δ BDE = Δ CDE
[same base DE and between //]

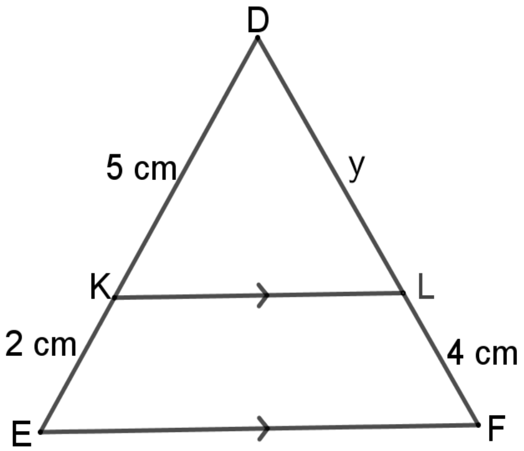
$$\frac{Area\ \Delta ADE}{Area\ \Delta BDE} = \frac{Area\ \Delta ADE}{Area\ \Delta CDE}$$

$$\therefore \ \frac{\boldsymbol{AD}}{\boldsymbol{DB}} = \frac{\boldsymbol{AE}}{\boldsymbol{EC}}$$



WORKED EXAMPLE 1:

In Δ DEF, KL//EF. Calculate the value of y.



Solution:

$\frac{DK}{KE} = \frac{DL}{LF}$ [prop theorem; $KL//EF$]

$\frac{5}{2} = \frac{y}{4}$

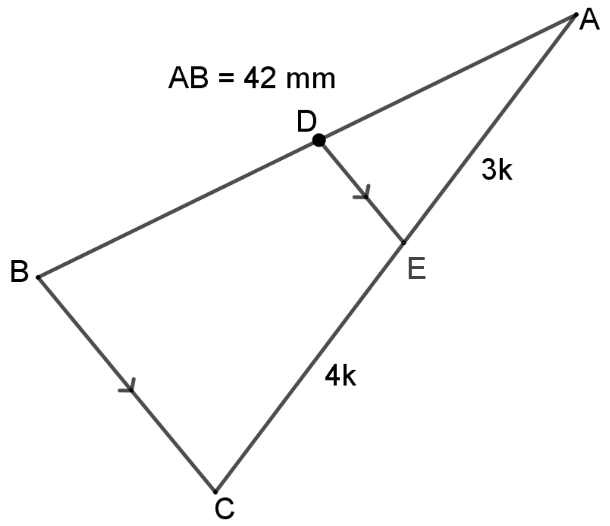
$2y = 20$

$y = 10\ cm$

WORKED EXAMPLE 2:

In Δ ABC, DE//BC, AB $= 42\ mm$ and AE: EC $= 3:4$

Determine the length of BD.



$\frac{BD}{AB} = \frac{CE}{AC}$[prop theorem; $BC//DE$]

$$\frac{BD}{42} = \frac{4k}{7k}$$

$$\frac{BD}{42} = \frac{4}{7}$$

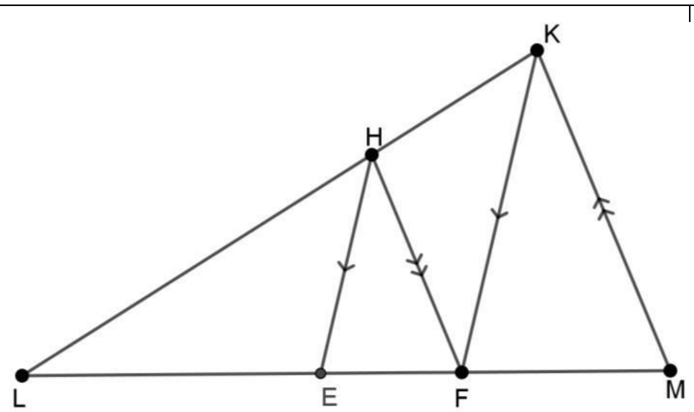
$$7\ BD = 168$$

$$BD = 24\ mm$$

WORKED EXAMPLE 3:

In Δ KLM, KM//DF, KF//DE and FE: EL $= 3:4$.

Determine LE: FM



Solution:

$\frac{KH}{HL} = \frac{MF}{LF} = \frac{MF}{7k}$[prop thm; $DF//KM$]

$\therefore \frac{FE}{EL} = \frac{MF}{LF}$[both$= \frac{KD}{DL}$]

$$\frac{3k}{4k} = \frac{MF}{7k}$$

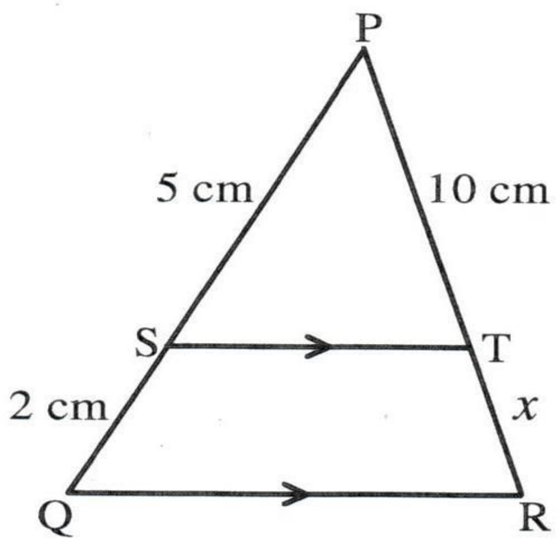
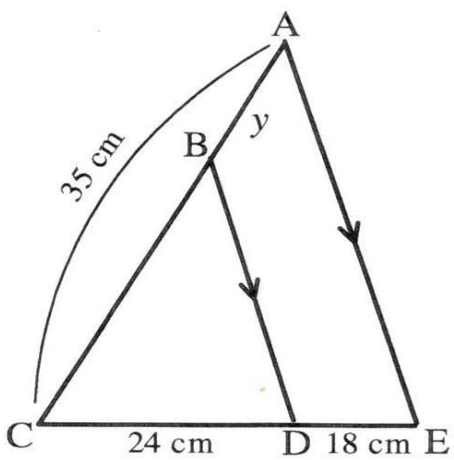
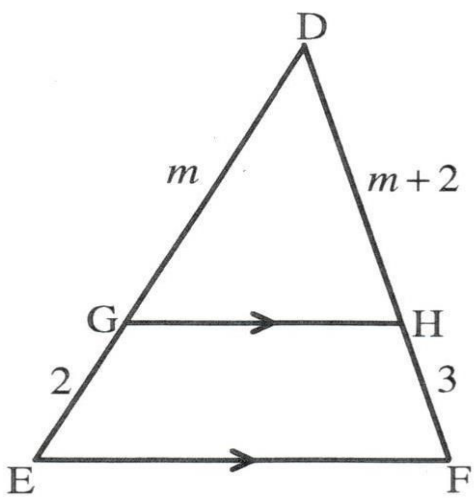
$$MF = \frac{21k^2}{4k} = \frac{21k}{4}$$

$$\therefore \frac{LE}{FM} = 4k \div \frac{21k}{4}$$

$$= \frac{4k}{1} \times \frac{4}{21k} = \frac{16k}{21k} = \frac{16}{21}$$

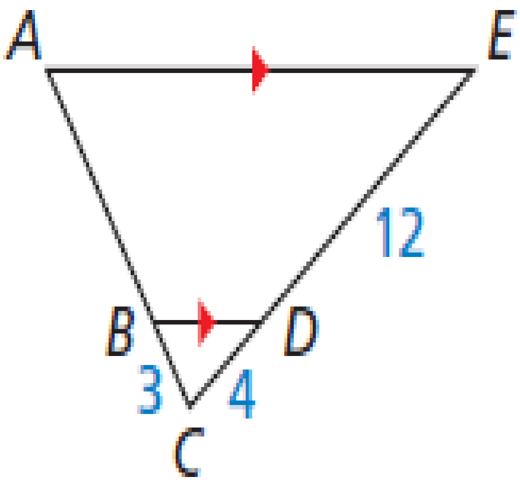
$$\therefore FE: FM = 16:21$$

HOMEWORK ACTIVITY

1	In ΔPQR , $ST//QR$. Calculate the value of x. $x = 4\ cm$ 
2	In ΔACE , $BD//AE$. Calculate the value of y. y =15 cm 
3	In ΔDEF , $GH//EF$. Calculate the value of m. $m = 4$ 

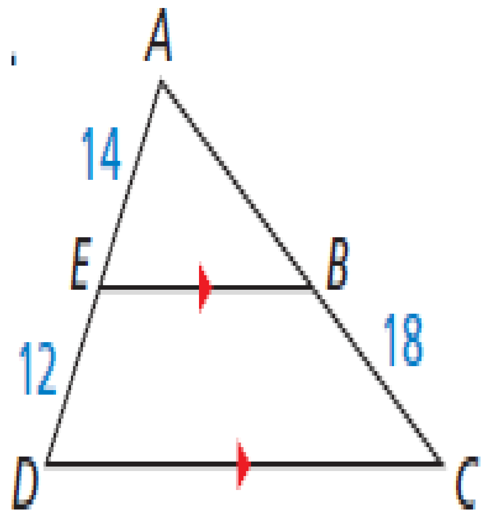
4	Calcualte the length AB.

a)



$\boldsymbol{AB = 9\ units}$

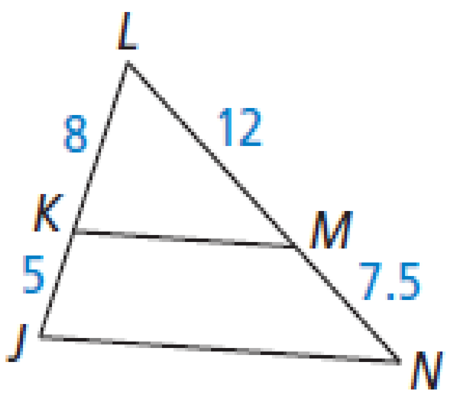
b)



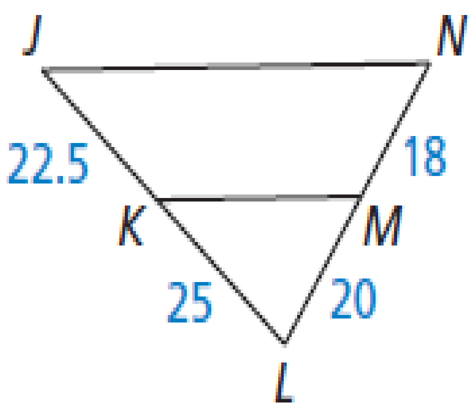
$\boldsymbol{AB = 21\ units}$

5	Determine whether $KM//JN$.

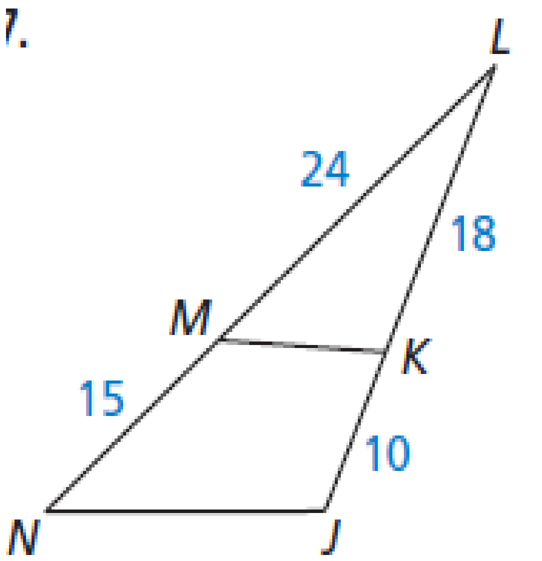
a)



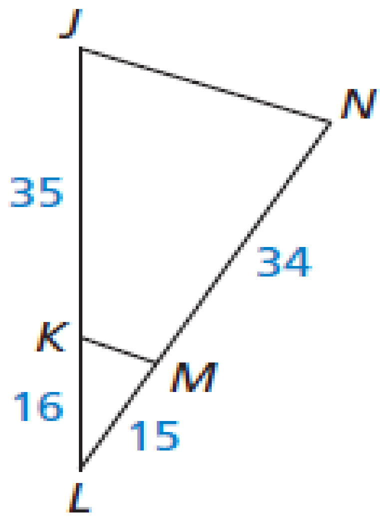
b)



c)

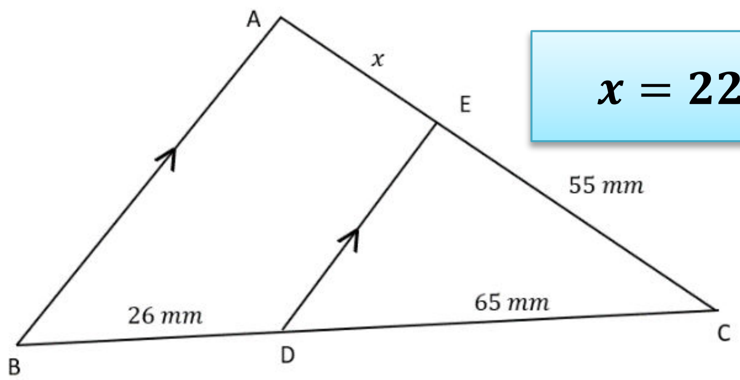


d)



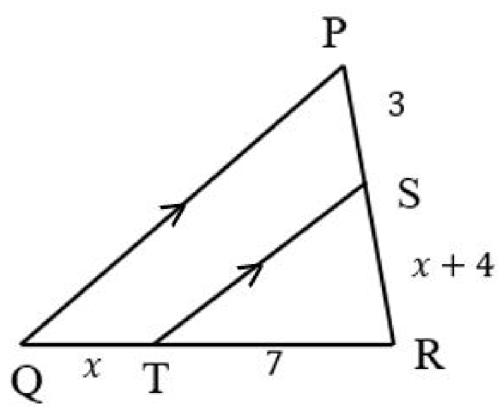
6 Calculate the length of sides labelled x and y in the following questions

a)



$$x = 22\ mm$$

b)



$$x = 3$$

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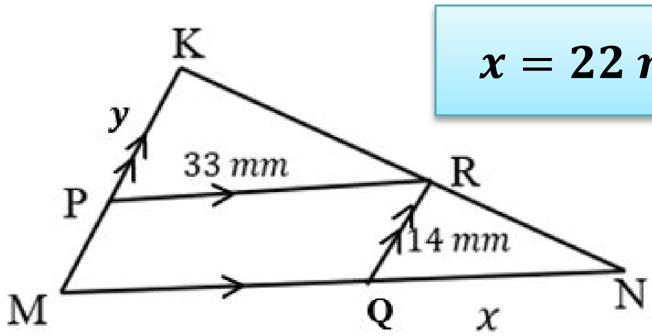
PMQR is a parallelogram PR=33mm, QR=14mm and KR : RN = 3 : 2

$x = 22\ mm; y = 21\ mm$



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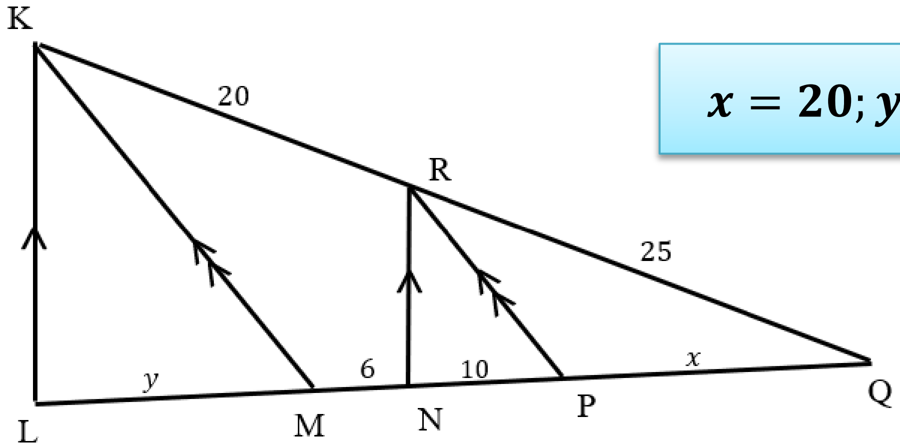
<tr><td>d)</td><td>

$x = 20; y = 18$



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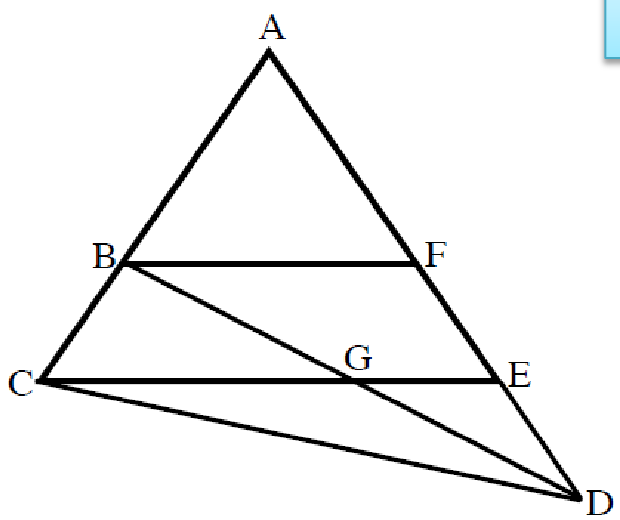
In $\Delta ACE, BF \parallel CE, \frac{BC}{AC} = \frac{3}{8}$ and $AE : ED = 4:3.$ Determine $DG : GB$.

2 : 1



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GRADE 12 – WORKSHEET 5

In the diagram, points D and E lie on sides AB and AC respectively of ΔABC such the DE//BC. Use Euclidean Geometry methods to prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$