GRADE 12 – WORKSHEET 7

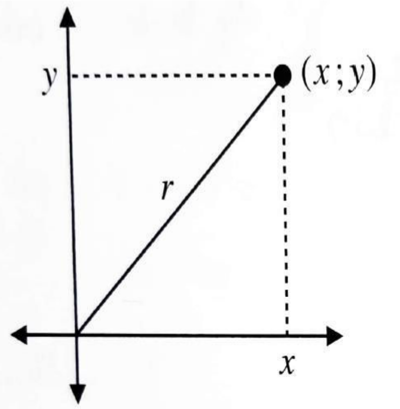
TRIGONOMETRY 1 – COMPOUND ANGLES IDENTITIES

OUTCOMES:

1. Compound angles identities

INTRODUCTION - OVERVIEW OF BASIC CONCEPTS

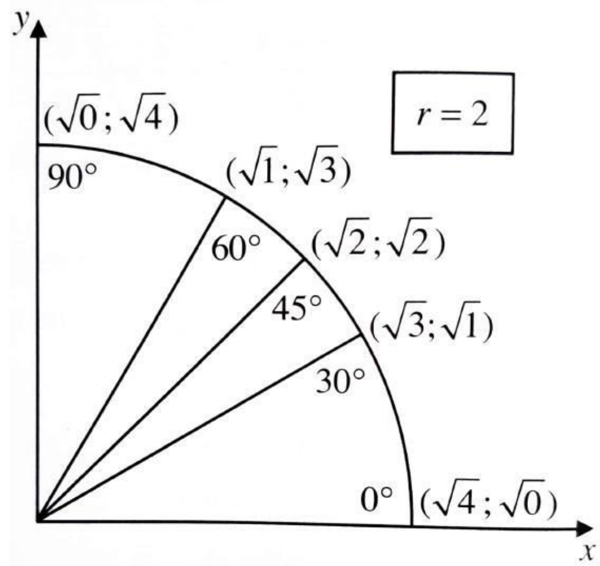
DEFINITIONS AND PYTHAGORUS:



$\boxed{x^2+y^2=r^2}$

$\boxed{\sin\theta=\frac{y}{r}}$ $\boxed{\cos\theta=\frac{x}{r}}$ $\boxed{\tan\theta=\frac{y}{x}}$

SPECIAL ANGLES:



REDUCTION AND CO-FUNCTION FORMULAE:

Second Quadrant:

- **sine function is positive**
- $\sin(180°-\theta)=\sin\theta$
- $\cos(180°-\theta)=-\cos\theta$
- $\tan(180°-\theta)=-\tan\theta$
- $\sin(90°+\theta)=\cos\theta$
- $\cos(90°+\theta)=-\sin\theta$

First Quadrant:

- **all functions are positive**
- $\sin(360°+\theta)=\sin\theta$
- $\cos(360°+\theta)=\cos\theta$
- $\tan(360°+\theta)=\tan\theta$
- $\sin(90°-\theta)=\cos\theta$
- $\cos(90°-\theta)=\sin\theta$

Third Quadrant:

- **tangent function is positive**
- $\sin(180°+\theta)=-\sin\theta$
- $\cos(180°+\theta)=-\cos\theta$
- $\tan(180°+\theta)=\tan\theta$

Fourth Quadrant:

- **cosine function is positive**
- $\sin(360°-\theta)=-\sin\theta$
- $\cos(360°-\theta)=\cos\theta$
- $\tan(360°-\theta)=-\tan\theta$

IDENTITIES:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

COMPOUND ANGLE FORMULAE:

1. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
2. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
3. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
4. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

WORKED EXAMPLE 1:

Expand using the compound angle identities:

a)	$\sin(x+y) = \sin x\cos y + \cos x\sin y$
b)	$\cos(\alpha + 10°) = \cos\alpha\cos 10° - \sin\alpha\sin 10°$
c)	$\cos(x + 60°) = \cos x\ \cos 60° - \sin x\ \sin 60°$ $= \cos x.\left(\frac{1}{2}\right) - \sin x.\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\cos x - \sqrt{3}\ \sin x}{2}$
d)	$\sin(3x - 45°) = \sin 3x\cos 45° + \cos 3x\sin 45°$ $\sin 3x.\left(\frac{\sqrt{2}}{2}\right) + \cos 3x.\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{\sqrt{2}\sin 3x - \sqrt{2}\ \cos 3x}{2}$

WORKED EXAMPLE 2:

Write the following as a single trigonometric ratio:

a)	$\cos\alpha\cos\beta + \sin\alpha\sin\beta = \boldsymbol{\cos(\alpha-\beta)}$
b)	$\sin x\cos 40° + \cos x\sin 40° = \boldsymbol{\sin(x+40°)}$
c)	$\cos 2x\sin 3x - \sin 2x\cos 3x$ $= \boldsymbol{\sin 3x\cos 2x - \cos 3x\sin 2x}$ $= \boldsymbol{\sin(3x-2x)}$ $= \boldsymbol{\sin x}$

WORKED EXAMPLE 3:

Simplify: $\sin(x+30°) - \sin(x-30°)$

<u>Solution:</u>

$$\begin{aligned}\sin(x+30°) - \sin(x-30°) &= \boldsymbol{\sin x\cos 30° + \cos x\sin 30° - (\sin x\cos 30° - \cos x\sin 30°)}\\ &= \sin x\cos 30° + \boldsymbol{\cos x\sin 30°} - \sin x\cos 30° + \boldsymbol{\cos x\sin 30°}\\ &= 2\cos x\,\boldsymbol{\sin 30°}\\ &= 2\cos x\left(\frac{1}{2}\right)\\ &= \cos x\end{aligned}$$

WORKED EXAMPLE 4:

Calculate the values of the following without the use of a calculator:

a)	$\cos 75° = \boldsymbol{\cos(30°+45°)}$ $= \cos 30°\cos 45° - \sin 30°\sin 45°$ $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$ $= \frac{\sqrt{6}-\sqrt{2}}{4}$
b)	$\sin 70°\sin 10° + \cos 10°\cos 70°$

	$= \cos 70° \cos 10° + \sin 70° \sin 10°$ $= \cos(70° - 10°)$ $= \cos 60°$ $= \frac{1}{2}$

HOMEWORK ACTIVITY

1	Expand using the compound angle identities:
a)	$\sin(x + 20°)$
b)	$\cos(2x + 10°)$
c)	$\sin(\alpha - 2\beta)$
d)	$\cos(2P - 40°)$
e)	$\cos(\alpha + 30°)$
2	Use the compound angle identities to simplify each expression to only one term.
a)	$\cos 2A \cos A - \sin 2A \sin A$
b)	$\sin 2A \cos A + \cos 2A \sin A$
c)	$\sin 2P \sin P - \cos 2P \cos P$
d)	$\sin 2P \cos P - \cos 2P \sin P$
e)	$\sin 50° \cos 10° - \cos 50° \sin 10°$
f)	$\cos 70° \sin 30° - \sin 70° \cos 30°$
3	Evaluate without using a calculator.
a)	$\sin 40° \cos 20° + \cos 40° \sin 20°$
b)	$\cos 40° \cos 20° - \sin 40° \sin 20°$

c)	$\cos 18° \cos 27° - \sin 18° \sin 27°$
d)	$\cos 75° \cos 15° + \sin 75° \sin 15°$
e)	$\sin 20° \cos 40° + \cos 20° \sin 40°$
f)	$\sin 70° \cos 25° - \sin 20° \sin 25°$
4	Prove the following identities:
a)	$\cos(45° - \theta) - \sin(45° - \theta) = \sqrt{2} \sin \theta$
b)	$\cos(A - 60°) + \cos(A + 60°) = \cos A$
c)	$\cos(P + Q) + \cos(P - Q) = 2 \cos P \cos Q$
d)	$\sin(A + 30°) - \sin(A - 30°) = \cos A$
e)	$\sin(\theta + 60°) + \cos(\theta + 150°) = 0$
f)	$\sin 15° = \dfrac{\sqrt{6} - \sqrt{2}}{4}$
g)	$\cos 15° = \dfrac{\sqrt{6} + \sqrt{2}}{4}$
h)	$\sin(60° + 45°) = \dfrac{\sqrt{6} + \sqrt{2}}{4}$
i)	$\cos 75° = \dfrac{\sqrt{6} - \sqrt{2}}{4}$