TRIGONOMETRY – DOUBLE ANGLE IDENTITIES

OUTCOMES:

1. Double angle identities

INTRODUCTION - OVERVIEW OF BASIC CONCEPTS

DOUBLE ANGLES: A double angle is an formed by adding an angle to itself, for example $\hat{\mathbf{A}} + \hat{\mathbf{A}} = 2\hat{\mathbf{A}}$; $\boldsymbol{\alpha} + \boldsymbol{\alpha} = 2\boldsymbol{\alpha}$; $\mathbf{10^\circ} + \mathbf{10^\circ} = \mathbf{20^\circ}$ e.t.c.

Although trigonometric identities can be challenging, it is always possible to earn some marks.

- Any $\tan\theta$ ratio should be written as $\frac{\sin\theta}{\cos\theta}$
- Any $\sin 2\theta$ ratio should be converted to $2\sin\theta\cos\theta$
- There is very often a ONE (1) that accompanies a $\cos 2\theta$ ratio. It is then advisable to substitute the $\cos 2\theta$ formula with the one that will cancel out the ONE (1).
- If your expression involves $\sin\theta$ and $\cos\theta$:
  it is advisable to use $\cos 2\theta = \cos^2\theta - \sin^2\theta$
- If your expression involves $\sin\theta$:
  it is advisable to use $\cos 2\theta = 1 - 2\sin^2\theta$
- If your expression involves $\cos\theta$:
  it is advisable to use $\cos 2\theta = 2\cos^2\theta - 1$

$$\mathbf{\sin 2A = 2\sin A\cos A}$$

$$\mathbf{\cos 2A = \cos^2 A - \sin^2 A}$$
$$\mathbf{\cos 2A = 2\cos^2 A - 1}$$
$$\mathbf{\cos 2A = 1 - 2\sin^2 A}$$

$$\sin^2 A + \cos^2 A = 1$$
$$\cos^2\mathrm{A} = 1 - \sin^2\mathrm{A}$$
$$\sin^2\mathrm{A} = 1 - \cos^2\mathrm{A}$$

WORKED EXAMPLE 1:
Simplify the following:

a)	$\cos 2x + 2\sin^2 x$
b)	$\frac{\sin 2\theta}{\sin\theta}$
c)	$\frac{\cos 2\mathrm{A} + 1}{2\cos\mathrm{A}}$

WORKED EXAMPLE 2:
Calculate the following without the use of a calculator.

a)	$2\sin 15° \cos 15°$

b)	$\sin 22{,}5\,° \cos 22{,}5°$
c)	$2\cos^2 15°$

WORKED EXAMPLE 3:

If $\mathbf{5\tan\alpha - 12 = 0}$ with $\boldsymbol{\alpha} \in [\mathbf{180°; 360}]$ and $\mathbf{5\cos\beta = -3}$ with $\mathbf{\sin\beta > 0}$ **,** determine the value of the following trigonometric ratios without the use of a calculator and with the aid of a diagram .

a)	$\sin(\alpha + \beta)$

b)	$\sin(\alpha - \beta)$
c)	$\cos(\alpha + \beta)$ $= \cos\alpha\cos\beta - \sin\alpha\sin\beta$
d)	$\sin 2\beta$ $= 2\sin\beta\cos\beta$

WORKED EXAMPLE 4:

Prove that $\frac{\sin 2x}{1+\cos 2x} = \tan x$

Solution:

WORKED EXAMPLE 5:

Prove: $\sin 3x = 3\sin x - 4\sin^3 x$

Solution:

WORKED EXAMPLE 6:

HOMEWORK ACTIVITY

1	If $25\sin\alpha - 7 = 0$, with $90° \leq \alpha \leq 270°$, and $5\cos\beta = 4$, with $180° \leq \beta \leq 360°$, Determine with the aid of a sketch the value of :
a)	$\tan\alpha$
b)	$\sin 2\beta$
c)	$\tan 2\beta$
d)	$\cos(\alpha - \beta)$
e)	$\sin(\alpha - \beta)$
2	Prove the following identities:
a)	$\dfrac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x$
b)	$\dfrac{1 - \sin 2x}{\sin x - \cos x} = \sin x - \cos x$
c)	$\dfrac{1 - \cos 2\beta}{\tan\beta} = \sin 2\beta$
d)	$(\sin x + \cos x)^2 = 1 + \sin 2x$
e)	$\dfrac{\sin\beta - \sin 2\beta}{\cos\beta - \cos 2\beta - 1} = \tan\beta$
f)	$\tan\theta + \dfrac{cos\theta}{\sin\theta} = \dfrac{2}{\sin 2\theta}$
g)	$\dfrac{\cos 2P}{\cos P - \sin P} = \cos P + \sin P$

h)	$\dfrac{\cos 2A + \cos A}{\sin 2A - \sin A} = \dfrac{\cos A + 1}{\sin A}$
i)	$\dfrac{\sin 2\theta + \cos 2\theta + 1}{\sin \theta + \cos \theta} = 2\cos\theta$
j)	$\dfrac{1 - \cos 2\theta}{\sin 2\theta} = \tan\theta$
k)	$\dfrac{1}{\cos 2x} + \dfrac{\sin 2x}{\cos 2x} = \dfrac{\sin x + \cos x}{\cos x - \sin x}$
l)	$\dfrac{\cos 2\beta + \cos \beta}{\sin 2\beta - \sin \beta} = \dfrac{\cos \beta + 1}{\sin \beta}$
m)	$\sin 3x = 3\sin x - 4\sin^3 x$