GRADE 12 – WORKSHEET 10

TRIGONOMETRY 4 – GRADE 11 BASICS

OUTCOMES:

1. Reduction formulae
2. Negative angles and angles greater than $360°$

INTRODUCTION - OVERVIEW OF BASIC CONCEPTS

Reduction formulae are used to **REDUCE** the trigonometric ratio of any angle to the trigonometric of an acute angle. The formulae we are going to use are:

Second Quadrant:	**First Quadrant:**
• **sine function is positive**	• **all functions are positive**
• $\sin(180° - \theta) = \sin\theta$	• $\sin(360° + \theta) = \sin\theta$
• $\cos(180° - \theta) = -\cos\theta$	• $\cos(360° + \theta) = \cos\theta$
• $\tan(180° - \theta) = -\tan\theta$	• $\tan(360° + \theta) = \tan\theta$
• $\sin(90° + \theta) = \cos\theta$	• $\sin(90° - \theta) = \cos\theta$
• $\cos(90° + \theta) = -\sin\theta$	• $\cos(90° - \theta) = \sin\theta$
Third Quadrant:	**Fourth Quadrant:**
• **tangent function is positive**	• **cosine function is positive**
• $\sin(180° + \theta) = -\sin\theta$	• $\sin(360° - \theta) = -\sin\theta$
• $\cos(180° + \theta) = -\cos\theta$	• $\cos(360° - \theta) = \cos\theta$
• $\tan(180° + \theta) = \tan\theta$	• $\tan(360° - \theta) = -\tan\theta$

WORKED EXAMPLE 1:

Simplify the following as far as possible:

$$\frac{\cos(90° + \theta)}{\sin(360° - \theta)}$$

NOTE:

Function values of $90° \pm \theta$ are the only function values that change the trigonometric function you are working with, when simplifying. When you are dealing with function values of $90° \pm \theta$, sine **CHANGES** to cosine and cosine changes to sine.

$90° + \theta$ (lies in the 2nd quadrant and the sign (+or-) of cosine is negative in the 2nd quadrant. You always use the original trigonometric function to determine the sign.

$360° - \theta$ (lies in the 4th quadrant and the sign of sine is negative in the 4th quadrant.)

$$\frac{-\sin\theta}{-\sin\theta}$$

$$= 1$$

WORKED EXAMPLE 2:

Simplify the following as far as possible:

a)	$\tan(\theta - 180°)$ $= \tan(\theta + 180°)$ – Add 360° $= \tan(180° + \theta)$ – rewrite in correct format $= \tan\theta$
b)	$\sin(-\theta - 90°)$ $= \sin(-\theta + 270°)$ – Add 360° $= \sin(270° - \theta)$ – rewrite in correct format $= -\cos\theta$

WORKED EXAMPLE 3:

Write the following as a function value of a positive acute angle.

a)	$\sin(-30°)$ $= -\sin 30°$ NOTE: $\sin(-\theta) = -\sin\theta$; quadrant 4 and only $\cos\theta$ is positive in quadrant 4.

b)	$\cos(-40°)$ $= \cos 40°$
c)	$\tan(-150°)$ – Add $360°$ $= \tan(210°)$ $= \tan(180° + 30°)$ $= \tan 30$
d)	$\cos 920°$ NOTE: because $920°$ is greater than $360°$, you may subtract $360°$twice in order to work with an angle in the interval $[0°; 360°]$. In other words, you can proceed as follows: $\cos 920° = \cos 200°$ - subtract $360°$ twice $= \cos 200°$ $= \cos(180° + 20°)$ $= -\cos 20°$

WORKED EXAMPLE 4:

If $\cos 20° = t$ determine the value(s) of the following in terms of t.

$\cos 20° = \frac{a}{h} = \frac{x}{r} = \frac{t}{1}$

$x^2 + y^2 = r^2$ (Pythagorus)

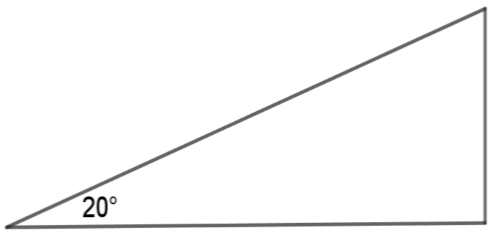
$t^2 + y^2 = 1^2$ (Pythagorus)

$y^2 = 1^2 - t^2$

$y = \sqrt{1^2 - t^2}$

$y = \sqrt{1 - t^2}$

x	a	t
y	o	$\sqrt{1 - t^2}$
r	h	1



a)	$\cos 160°$

	$= \cos(180° - 20°)$ – use the reduction formulae $= -\cos 20° \ \ (\cos 20° = t)$ $= -\left(\frac{t}{1}\right)$ $= -t$
b)	$\cos(-200°)$ $= \cos(-200° + 360°)$ $= \cos(160°)$ $= \cos(180° - 20°)$ $= -\cos 20$ $= -\left(\frac{t}{1}\right)$ $= -t$
c)	$\sin 250°$ $= \sin(180° + 70°)$ – 3rd quadrant $= -\sin 70°$ $= -\sin(90° - 20°)$ $= -\cos 20°$ $= -\left(\frac{t}{1}\right)$ $= -t$
d)	$\sin 20°$ $= \frac{y}{r}$ $= \frac{\sqrt{1 - t^2}}{1}$
e)	$\tan 20°$

	$= \dfrac{y}{x}$ $= \dfrac{\sqrt{1-t^2}}{t}$

HOMEWORK ACTIVITY

1	Simplify the following: *x is an acute angle*
a)	$\dfrac{3\cos(180° + x).\sin(360° - x)}{\sin(360° + x).\cos(180° - x)}$ $\dfrac{(-3\cos x)(-\sin x)}{(\sin x.)(-\cos x)} = -3$
b)	$\dfrac{\cos(180° + x).\tan(180° - x).\sin(180° + x)}{\sin(180° - x).\sin x}$
c)	$\dfrac{\sin(360° - x).\tan(180° - x).\cos(360° - x)}{\sin(180° - x).\sin(180° + x)}$
d)	$\dfrac{\cos^2(180° - x).\tan^2(360° + x)}{\sin^2(180° - x)}$
e)	$\dfrac{\sin^2(180° + \theta) + \cos^2(180° - \theta)}{\tan(180° - \theta).\cos(360° - \theta)}$
f)	$\dfrac{\sin(\theta - 360°)\sin(90° - \theta)\tan(-\theta)}{\cos(90° + \theta)}$ $= \dfrac{\sin\theta.\cos\theta.-\tan\theta}{-\sin\theta} =$
g)	$\dfrac{\sin(180° - x) - 2\cos(90° - x).\cos x}{2\cos^2(360° + x) - \cos(-x)}$

h)	$\dfrac{\sin(270+x).\tan(-x)-\cos(180°+x).\cos(90+x)}{\sin^2(180°-x)+\cos(360°-x)+\cos^2(180+x)}$
2	Rewrite each ration as a ratio of an acute angle:
a)	$\sin 122°$
b)	$\tan 145°$
c)	$\cos 230°$
3	Rewrite each ratio as a ratio of a positive acute angle:
a)	$\sin(-112°)$
b)	$\tan(-125°)$
c)	$\cos(-292°)$
4	If $\cos 20° = p$, find these ratios in terms of p:
a)	$\cos 160°$
b)	$\sin(-70°)$
c)	$\cos 200°$
d)	$\sin 340°$
5	If $\cos 52 = k$, find each ration in terms of k.
a)	$\cos 128$
b)	$\sin 38°$
c)	$\tan 218°$
d)	$\sin 308°$
e)	$\cos 232°$

f)	$\tan(-142)$