TRIGONOMETRY 5 – GRADE 11 BASICS

OUTCOMES:

1. Sine rule
2. Cosine rule
3. Area rule

INTRODUCTION - OVERVIEW OF BASIC CONCEPTS

If you have enough information about the sides and angles of any triangle, you can use the sine rule or the cosine rule to find the other sides and angles.

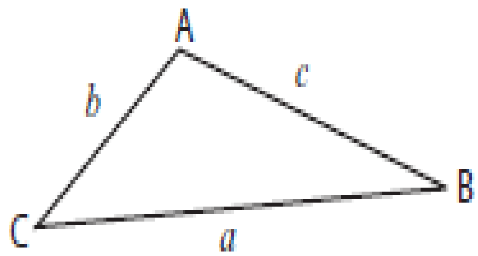
SINE RULE:

In triangle ABC:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

We can also use the sine rule in this form:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin c}$$



To use the **SINE RULE** you need to know at least one side and its matching opposite angle and one more side or angle.

COSINE RULE:

In any triangle ABC:

If you choose to use angle A, then

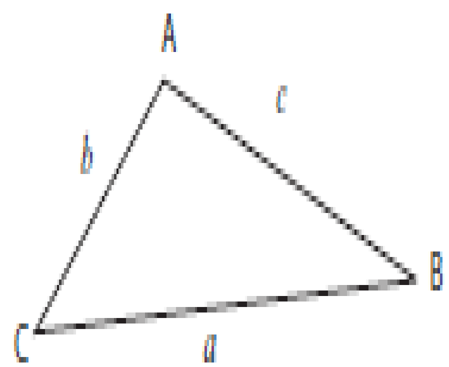
$$a^2 = b^2 + c^2 - 2bc\cos A$$

If you choose to use angle B, then

$$b^2 = a^2 + c^2 - 2ac\cos B$$

If you choose to use angle C, then

$$c^2 = a^2 + b^2 - 2ab\cos C$$

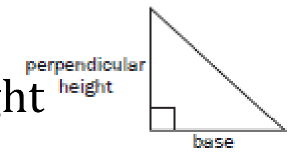


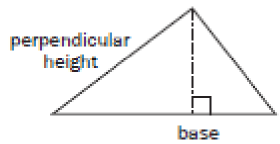
You apply the COSINE RULE if you are given the values of:

- Two sides and the included angle OR
- Three sides of a triangle

AREA RULE:

Area of a right-angled triangle:





Area $\Delta = \frac{1}{2}\text{base} \times \text{perpendicular height}$

Area $\Delta = \frac{1}{2}\text{bh}$

There is a formula that works to find the AREA of any triangle, even if we do not know the perpendicular height.

The area of any ΔABC is half the product of two sides and sine of the included angle

So if you choose to use angle A, then:

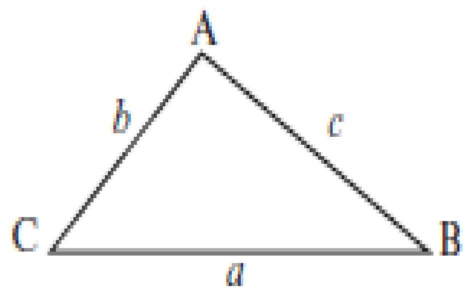
Area $\Delta\text{ABC} = \frac{1}{2}bc\sin A$

So if you choose to use angle B, then:

Area $\Delta\text{ABC} = \frac{1}{2}ac\sin B$

So if you choose to use angle C, then:

Area $\Delta\text{ABC} = \frac{1}{2}ab\sin C$

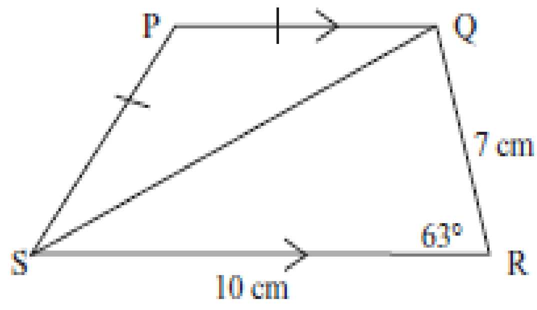


To find the AREA of any triangle, you need to know the lengths of two sides and the size of the angle between the two sides (included angle)

WORKED EXAMPLE 1:

PQRS is a trapezium with PQ//SR.
$\text{PQ} = \text{PS}$, $\text{SR} = 10$ cm, $\text{QR} = 7$ cm, $\hat{\text{R}} = 63°$.

Calculate:

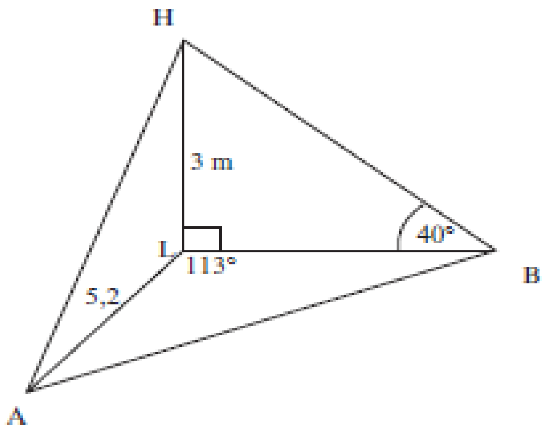


a)	SQ In ΔQSR, you know two sides and the included angle, so use the cosine rule. $SQ^2 = 7^2 + 10^2 - 2(7)(10)\cos 63°$ $SQ^2 = 85{,}44\ldots$ $SQ = \sqrt{85{,}44..}$ $SQ = 9{,}24\ cm$
b)	PS In Δ PQS, you know that $\mathrm{PQ} = \mathrm{PS}$ and you worked out that $\mathrm{SQ} = 9{,}24\ \mathrm{cm}$. THINK ABOUT THE QUESTION FIRST If you can find $\widehat{\mathrm{P}}$ then you can use the sine rule to find PS. To find $\widehat{\mathrm{P}}$, you need to first find $\mathrm{P\hat{Q}S}$ or $\mathrm{P\hat{S}Q}$. $\mathrm{P\hat{Q}S} = \mathrm{P\hat{S}Q}$ (alternate angles, $\mathrm{PQ//SR}$) Now you can work out a the value for $\mathrm{Q\hat{S}R}$. In $\Delta\mathrm{QSR}$, you know three sides and $\widehat{\mathrm{R}}$ So it is easier to use the sine rules to find $\mathrm{Q\hat{S}R}$. $\dfrac{\sin Q\hat{S}R}{7} = \dfrac{\sin 63°}{9{,}24}$ $\sin Q\hat{S}R = \dfrac{7\sin 63°}{9{,}24} = 0{,}675004$ $\therefore Q\hat{S}R = \sin^{-1}(0{,}675004)$ $= 42{,}45°$ $P\hat{Q}S = Q\hat{S}R = 42{,}45°$ (alternate angles, $PQ//SR$) $P\hat{Q}S = P\hat{S}Q = 42{,}45°$ (base angles of isosceles, Δ)

	$\therefore \hat{P} = 180° - (42,45° + 42,45°)$ $\hat{P} = 95,1°$ Now we can find PS using the sine rule and $\hat{P}$. In ΔPQS: $\frac{PS}{\sin 42,45°} = \frac{9,24}{\sin 95,1°}$ $PS = \frac{9,24}{\sin 95,1°} \times \sin 42,45°$ $PS = 6,26\ cm$
c)	Area of quadrilateral PQRS (correct to 2 decimal places). To find the areas of $PQRS$, find the area of the two triangles and add them together. To find the area of ΔPQS, use $\hat{P} = 95,1°$ and $PS = PQ = 6,26\ cm$. Area $\Delta PQS = \frac{1}{2} qs \sin \hat{P}$ Area $\Delta PQS = \frac{1}{2}(6,26)(6,26) \sin 95,1°$ Area $\Delta PQS = 19,52 cm^2$ To find the area of ΔRQS, use $\hat{R} = 63°$; $QR = 7\ cm$ and $SR = 10\ cm$. Area $\Delta RQS = \frac{1}{2}(7)(10) \sin 63°$ Area $\Delta RQS = 31,19\ cm^2$ $\therefore Area\ PQRS = 19,52 + 31,19 = 50,71\ cm^2$

WORKED EXAMPLE 2:

A, B and L are points in the same horizontal pane, HL is a vertical pole of length 3 m, $AL = 5{,}2$ m and the angle $A\hat{L}B = 113°$ and the angles of elevation of H from B is $40°$.



a) Calculate the length of LB.

In ΔHLB, $\tan 40° = \frac{3}{LB}$ (ΔHLB is right-angled, so use trig ratio)

$$LB = \frac{3}{\tan 40°}$$

$$LB = 3{,}5752\ \ldots$$

$$LB = 3{,}58\text{ m}$$

b) Hence, or otherwise, calculate the length of AB.

ΔABL is not right-angled. You have two sides and included angle, so use cosine rule.

$$AB^2 = AL^2 + BL^2 - 2(AL)(BL).\cos L$$

$$AB^2 = (5{,}2)^2 + (3{,}58)^2 - 2(5{,}2)(3{,}58).\cos 113°$$

$$AB^2 = 54{,}40410\ \ldots\text{ m}^2$$

$$AB = 7{,}38\text{ m}$$

c) Determine the area of ΔABL.

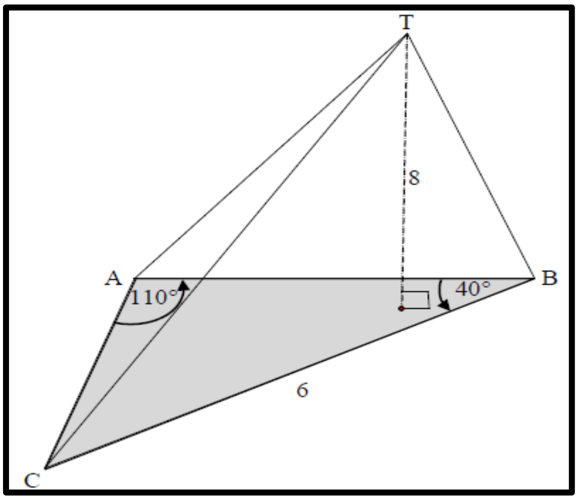
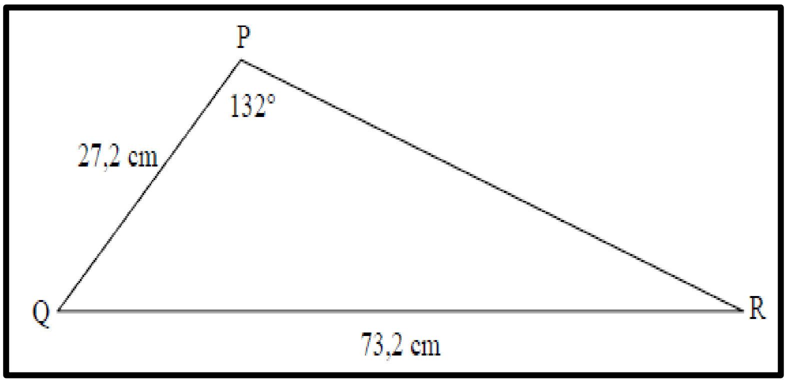
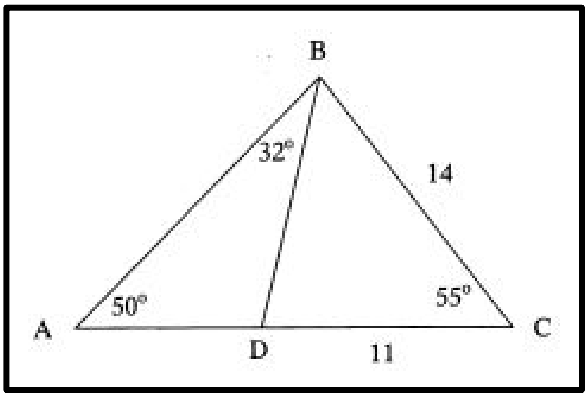
Area $\Delta ABL = \frac{1}{2} AL \times BL \times \sin A\hat{L}B$

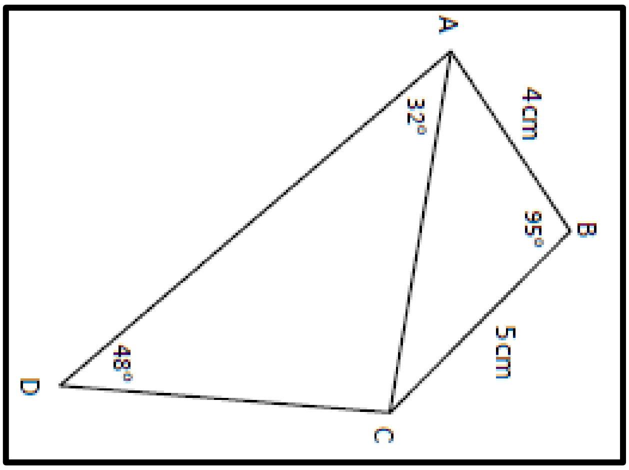
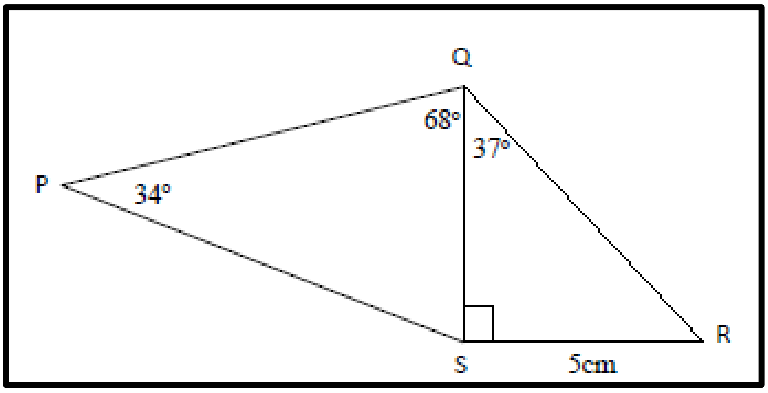
$$= \frac{1}{2}(5{,}2)(3{,}58)\sin 113°$$

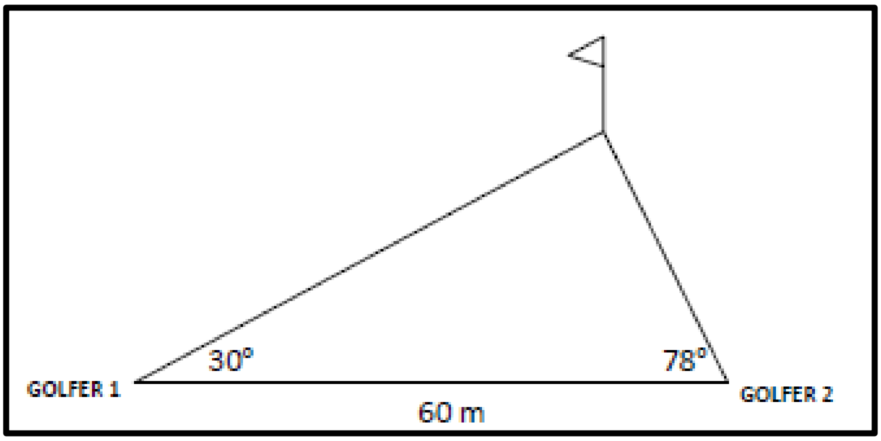
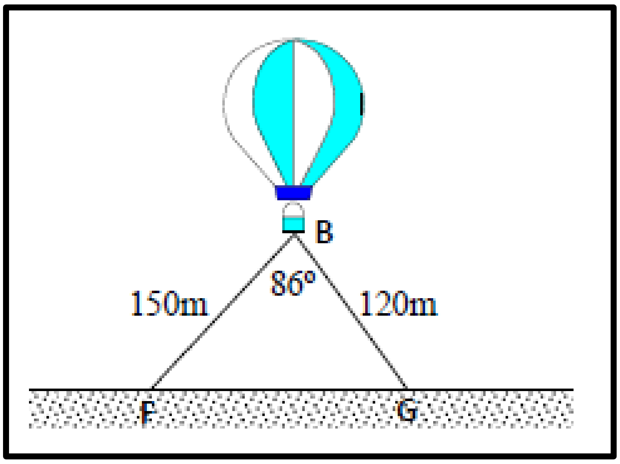
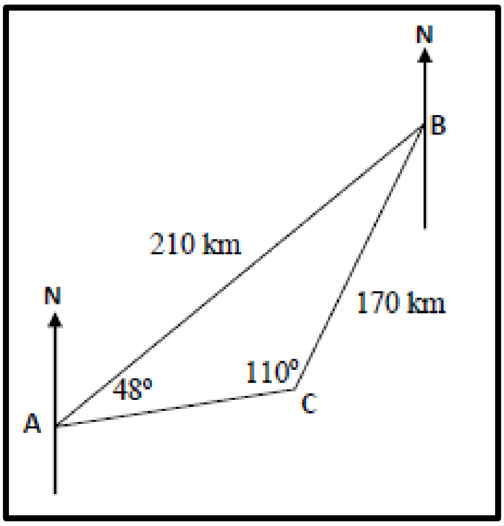
$$= 8{,}56805\ \ldots.$$

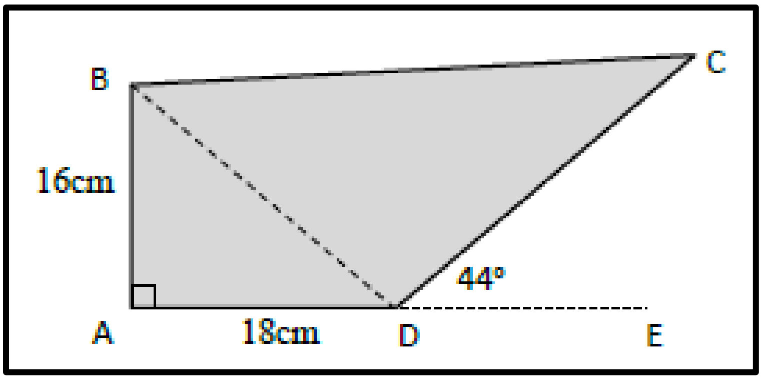
	$= 8{,}57 \text{ m}^2$

HOMEWORK ACTIVITY

1	In the diagram, the base of the pyramid is an obtuse-angled ΔABC with $\hat{A} = 110°$, $\hat{B} = 40°$ and $BC = 6$ metres. The perpendicular height of the pyramid is 8 metres. 	
a)	Calculate the length of AB.	$AB = 3{,}19\ m$
b)	Calculate the area of the base, that is ΔABC.	$A = 6{,}15\ m^2$
2	In ΔPQR. $\hat{P} = 132°$, $PQ = 27{,}2$ cm and $QR = 73{,}2$ cm. 	
a)	Calculate the size of $\hat{R}$.	$R = 16{,}03°$
b)	Calculate the area of ΔPQR.	$A = 527{,}10\ cm^2$
3		

a)	Calculate the length BD.
b)	Calculate the length AD.
c)	Calculate the area of triangle ABC.
4	
a)	Calculate the length of AC.
b)	Calculate the size of $B\hat{A}C$.
c)	Write down the size of $A\hat{C}D$.
d)	Calculate the length of AD.
e)	Calculate the area of the quadrilateral ABCD.
5	 Calculate the length of PS.
6	Two golfers are aiming for the green. The golfers are 60 m apart and the angles are as shown in the diagram. What distance will

	each golfer have to hit the ball in order to reach the pin? 
7	A hot air balloon B is fixed to the ground at F and G by 2 ropes 120 m and 150 m long. If $A\hat{B}G$ is 86°, how far apart are F and G? 
8	The diagram shows the path of an aircraft from A to B to C. 
a)	Write down the size of $A\hat{B}C$.
b)	Calculate the distance of AC.

9	The diagram below a steel plate ABCD.	
a)	Calculate the length of BD correct to 1 decimal place.	
b)	Find the size of angle $\mathrm{B\widehat{D}C}$ correct to the nearest degree.	
c)	Calculate the length of BC given that $\mathrm{DC} = 25\ \mathrm{cm}$.	