

ANALYTICAL GEOMETRY 1 – GRADE 10 & 11 REVISION

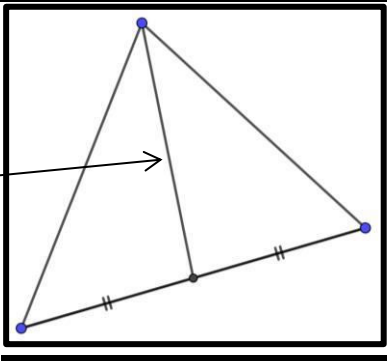
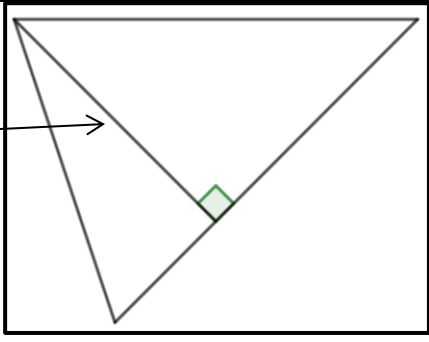
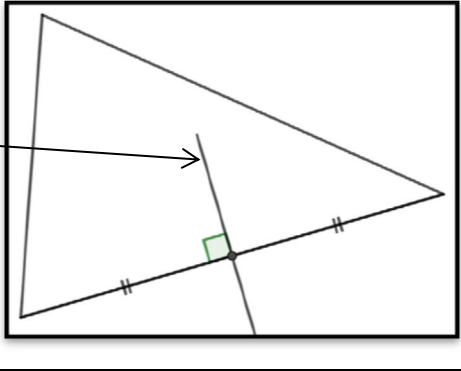
OUTCOMES:

1. Pythagoras, Distance, gradient, midpoint, inclination formula. If lines $\perp$, then product of their gradient is -1
2. The equation of a line through two given points;
3. The equation of a line through one point and parallel or perpendicular to a given line; find the perpendicular bisector.
4. The inclination (θ) of a line, where $m = \tan\theta$ is the gradient of the line ($0^\circ \leq \theta \leq 180^\circ$)
5. Properties of quadrilaterals.

IMPORTANT TERMINOLOGY

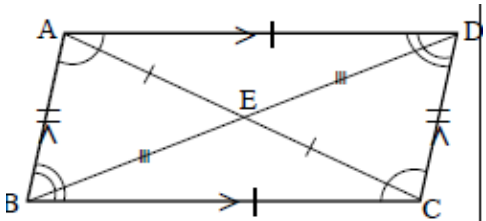
1. If two lines AB and DC are parallel, then their gradients are equal
 $\therefore \text{gradient}_{AB} = \text{gradient}_{DC}$ if $AB \parallel DC$.
2. If two lines AB and DC are perpendicular, then the product of the gradients equal -1 .
 $\therefore \text{gradient}_{AB} \times \text{gradient}_{DC} = -1$ if $AB \perp DC$
3. Points that are **collinear** lie on the same line. The gradient between each pair of points is the same. For example, if points A, B and C are collinear, then:
 $\text{gradient}_{AB} = \text{gradient}_{BC} = \text{gradient}_{AC}$
4. The inclination of a line is the angle formed with the horizontal in an anti-clockwise direction. On the Cartesian plane, the inclination of a line is calculated by finding the **angle formed at the x-axis** measured in anti-clockwise direction.
5. Bisects means to divide (a line, angle, or shape) into two exactly equal parts
6. A diagonal straight line joining two opposite corners of a square, rectangle, or other straight-sided shape

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A median is a line from vertex to midpoint of opposite side.	
An Altitude is a line from any vertex perpendicular to the opposite side.	
A perpendicular bisector is a line passes through the midpoint and is perpendicular to the line it is passing through.	

IDENTIFY AND CLASSIFY QUADRILATERALS

A quadrilateral is a 2D plane with four straight lines, joined together to form **FOUR** corners. Quadrilaterals can be identified by the **SIZES** of their **ANGLES OR LENGTHS** of their sides. The **SIX** different quadrilaterals are summarised below.

Classification of Quadrilateral	Properties	Drawing
Parallelogram	- Both pairs of opposite sides parallel and equal - opposite angles equal	

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	• Diagonals bisect each other	
Rhombus	• Diagonals bisect at right angles (90°) • Diagonals bisect the opposite angles • All sides are equal in length • Diagonals bisect each other	
Rectangle	• Both pairs of opposite sides parallel and equal • Diagonals are equal in length • All interior angles equal to 90°	Rectangle
Square	• Both pairs of opposite sides parallel • All sides equal in length • Diagonals bisect at right angles (90°) • All interior angles equal to 90° • Diagonals bisect interior angles (each bisected angle equals 45°)	

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Kite	• Adjacent pairs of sides are equal in length • The longer diagonal bisects the opposite angles • The longer diagonal bisects the other diagonal • The diagonals intersect at right angles(90°)	
Trapezium	• One pair of opposite sides parallel	

INTRODUCTION - OVERVIEW OF BASIC CONCEPTS

DISTANCE BETWEEN TWO POINTS:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

THE COORDINATES OF THE MIDPOINT OF A LINE SEGMENT:

$$M(x; y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

THE GRADIENT OF A LINE:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

THE EQUATION OF A LINE:

$$y = mx + c \text{ or } y - y_1 = m(x - x_1)$$

THE INCLINATION OF A LINE:

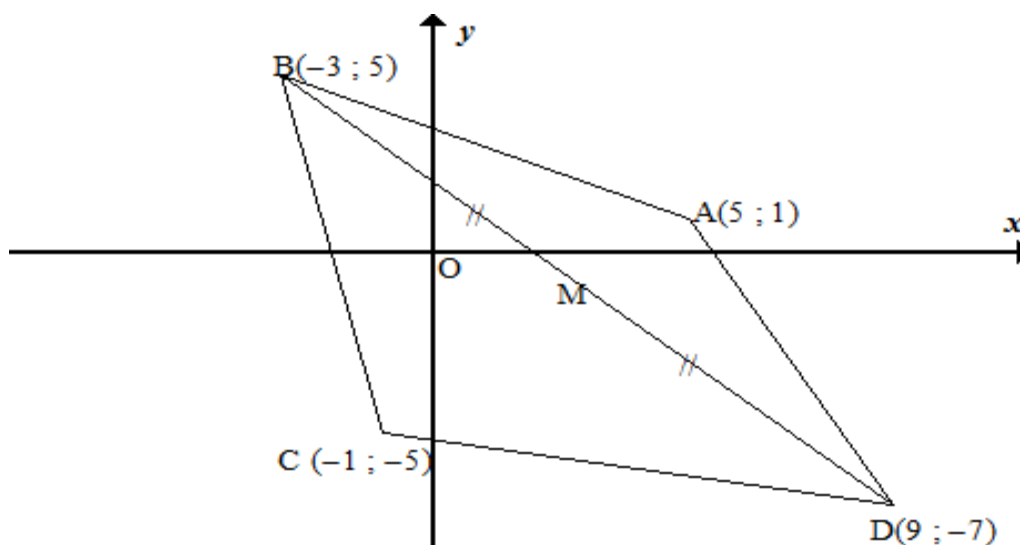
$$m = \tan \theta$$

θ is called the angle of inclination

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WORKED EXAMPLE 1:

ABCD is a quadrilateral with vertices A(5 ; 1), B(-3 ; 5), C(-1 ; -5) and D(9 ; -7).



a)	Calculate the gradient of AC. $m_{AC} = \frac{y_C - y_A}{x_C - x_A} m_{AC} = \frac{-5 - 1}{-1 - 5} m_{AC} = \frac{-6}{-6}$ $m_{AC} = 1 m_{AC} = \frac{-5 - 1}{-1 - 5} m_{AC} = \frac{-6}{-6} m_{AC} = 1$
b)	Determine the equation of AC in the form $y = mx + c$ $y - y_1 = m(x - x_1)$ $y - 1 = 1(x - 5)$ $y - 1 = x - 5$ $y = x - 4$
c)	Hence, or otherwise, show that the midpoint M of BD lies on AC. Midpoint of BD = $\left(\frac{x_B + x_D}{2}; \frac{y_B + y_D}{2} \right)$ Midpoint of BD = $\left(\frac{-3 + 9}{2}; \frac{5 - 7}{2} \right)$

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	Midpoint of BD = $(3; -1)$ The line AC is $y = x - 4$ [substitute midpoint of BD into equation, if LHS = RHS, then point lies on AC] Midpoint of BD = $(3; -1)$ LHS: $y = -1$ RHS: $x - 4 = 3 - 4 = -1$ LHS = RHS, $\therefore$ point M lies on line AC. $\therefore$ M lies on AC
d)	Show that $\widehat{AMB} = 90^\circ$. If the $m_{AM} \times m_{BM}$ is equal to negative one, then the $\widehat{AMB} = 90^\circ$. $m_{AM} = \frac{5-3}{1-(-1)} = 1$ [You can also use the gradient of AC because AC is a straight line.] $m_{MB} = \frac{-1-5}{3+3} = -1$ $m_{AM} = m_{AC}$ (collinear) $m_{AM} \times m_{BM} = -1, \therefore \widehat{AMB} = 90^\circ$.
e)	Calculate the area of $\triangle ABC$. $A = \frac{1}{2} b \perp h$ Distance AC = $\sqrt{72}$ Distance BM = $\sqrt{72}$ Area = $\frac{1}{2} \times \sqrt{72} \times \sqrt{72}$ Area = 36 units^2 $AC = \sqrt{(1 - (-5))^2 + (5 - (-1))^2} = 6\sqrt{2}$

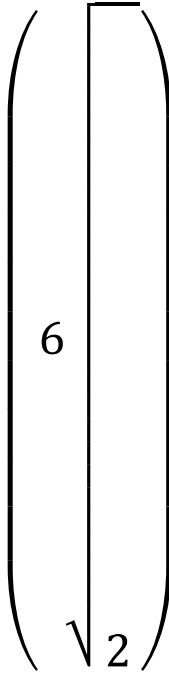
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$$BD = \sqrt{(5 - (-7))^2 + (-3 - 9)^2} = 12\sqrt{2}$$

$$\therefore BM = \frac{1}{2}(12\sqrt{2}) = 6\sqrt{2} \text{ (M is the midpoint)}$$

$$\therefore \text{Area} = \frac{1}{2} \cdot AC \cdot BM$$

$$\therefore \text{Area} = \frac{1}{2}(6\sqrt{2})$$



$$= 36 \text{ units}^2$$

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f)	Find the equation of the perpendicular bisector of CD. STEP 1: Determine the midpoint of CD $M(4; -6)$ STEP 2: Determine the gradient of CD $m_{CD} = -\frac{1}{5}$ STEP 3: Determine the gradient of Bisector $m_{CD} \times m_{BI} = -1$ $\therefore m_{BI} = 5$ STEP 4: Give the answer in the form $y = mx + c$ $m_{CD} = \frac{-5 - (-7)}{-1 - 9} = \frac{2}{-10} = -\frac{1}{5}$ $m_{CD} \times m_{BI} = -1$ $-\frac{1}{5} \times m_{BI} = -1$ $\therefore m_{BI} = 5$ $M_{CD} = \left(\frac{-1 - 5}{2}; \frac{-7 + 9}{2} \right)$ Midpoint of CD = $(-3; 1)$ $y - y_1 = m(x - x_1)$ Substitute midpoint into formula $y + 6 = 5(x - 4)$ $y + 6 = 5x - 20$ $y = 5x - 26$
g)	Find the coordinate of the points K such that BCDK is a parallelogram (i.e. if opposite sides are parallel to each other) Let $K = (x_K; y_K)$

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BCDK is a parallelogram]

$$\therefore m_{BC} = m_{DK}$$

$$\frac{y_B - y_C}{x_B - x_C} = \frac{y_K - y_D}{x_K - x_D}$$

$$\frac{5 + 5}{-3 + 1} = \frac{y_K + 7}{x_K - 9}$$

$$-5 = \frac{y_K + 7}{x_K - 9}$$

$$-5(x_K - 9) = y_K + 7$$

$$-5x + 45 = y + 7 \text{ equation 1}$$

$$\therefore m_{DC} = m_{BK}$$

$$\frac{y_C - y_D}{x_C - x_D} = \frac{y_K - y_B}{x_K - x_B}$$

$$\frac{-5 + 7}{-1 - 9} = \frac{y_K - 5}{x_K + 3}$$

$$-\frac{1}{5} = \frac{y_K - 5}{x_K + 3}$$

$$-1(x + 3) = 5(y - 5)$$

$$-x - 3 = 5y - 25 \text{ equation 2}$$

Make x the subject of the formula of equation 2. You can use equation 1 also.

$$-x = 5y - 22$$

$$x = 22 - 5y \text{ equation 3}$$

Substitute $x = 22 - 5y$ into equation 1

$$-5(22 - 5y) + 45 = y + 7$$

$$-110 + 25y + 45 = y + 7$$

$$24y = 72$$

$$\therefore y_K = 3$$

Substitute $y_K = 3$ into equation 3

$$x = 22 - 5(3)$$

$$\therefore x_K = 7 \therefore K(7 ; 3)$$

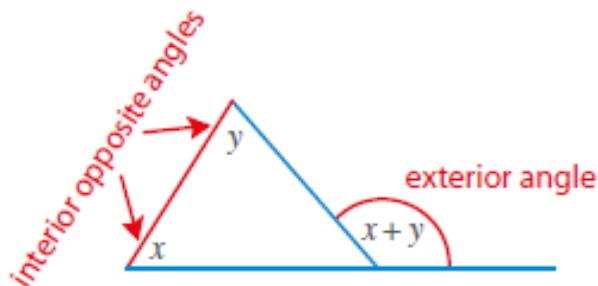
h) Find the size of the angle $\hat{C}BD$ correct to one decimal place.

Triangles have two important properties which are true for all triangles:

- The sum of the interior angles is equal to 180° .

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- The exterior angle of a triangle is equal to the sum of the two interior opposite angles.



Use gradient of BC

$$m_{BC} = \frac{y_B - y_C}{x_B - x_C}$$

$$m_{BC} = \frac{5 - (-5)}{-3 - (-1)}$$

$$m_{BC} = \frac{10}{-2}$$

$$m_{BC} = -5$$

Use gradient of BD

$$m_{BD} = \frac{y_B - y_D}{x_B - x_D}$$

$$m_{BD} = \frac{5 - (-7)}{-3 - 9}$$

$$m_{BD} = \frac{12}{-12}$$

$$m_{BD} = -1$$

Angle of inclination

$$\tan \theta = m_{BC}$$

$$RA = \tan^{-1}(5)$$

$$RA = 78,69^\circ$$

$$\theta = 180 - 78,69^\circ$$

$$\theta = 101,31^\circ$$

Angle of inclination of BD

$$\tan \alpha = m_{BD}$$

$$RA = \tan^{-1}(1)$$

$$RA = 45^\circ$$

$$\alpha = 180 - 45^\circ$$

$$\alpha = 135^\circ$$

The exterior angle of a triangle is equal to the sum of the two interior opposite angles.

$$\hat{C}BD = \alpha - \theta$$

$$\hat{C}BD = 135^\circ - 101,31^\circ$$

$$\hat{C}BD = 33,69^\circ \therefore \hat{C}BD = 135 - 10,31$$

$$\therefore \hat{C}BD = 135 - 10,31$$

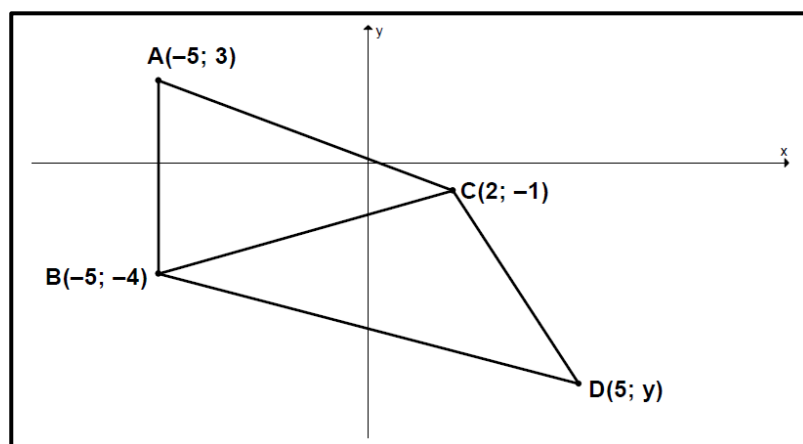
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$$\therefore \widehat{CBD} = 3$$

, 69°

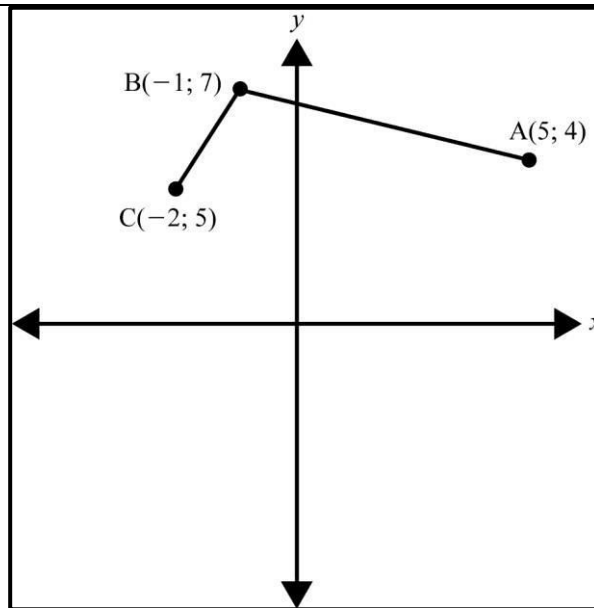
HOMEWORK ACTIVITY

- 1 In the diagram below, $A(-5; 3)$, $B(-5; -4)$, $C(2; -1)$ and $D(5; y)$ are points in the Cartesian Plane.



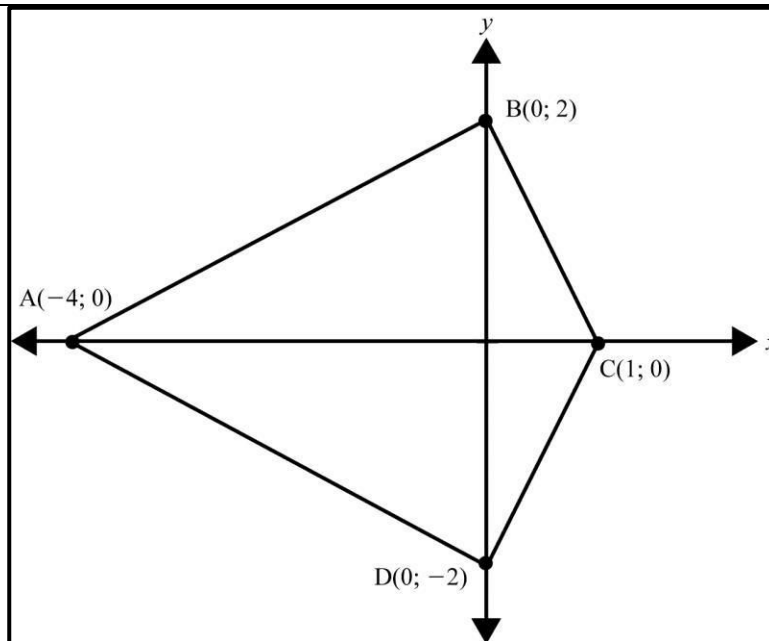
- a) Determine the gradient of BC .
 - b) Determine the length of AB .
 - c) The co - ordinates of M , the midpoint of BC .
 - d) Calculate the gradient of CD is $-\frac{7}{3}$, determine the value of y .
 - e) Prove that $\triangle BCD$ is a right angled triangle.
 - f) Determine the co - ordinates of $E(x; y)$ if quadrilateral $AECB$ is a parallelogram.
- 2 $M(-\frac{3}{2}; y)$ is the midpoint of a line segment AB . The co - ordinate of A and B are $(k; -4)$ and $(5; 10)$ respectively. Determine the values of k and y .
- 3 The points $A(5;4)$, $B(-1;7)$ and $C(-2;5)$ are shown below.

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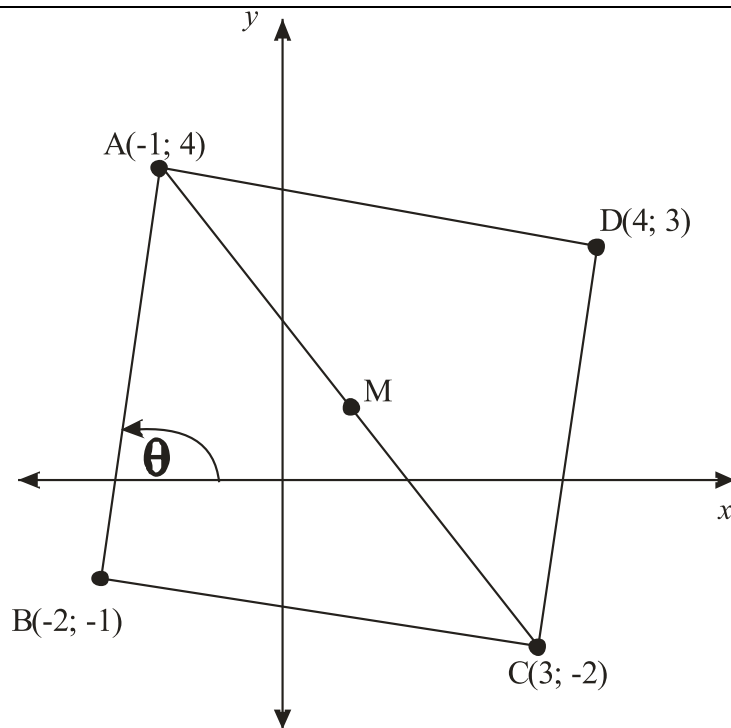
a)	Find the value of a if D is the point $(8;a)$ and $AB \parallel CD$.
b)	Find the equation of AB.
c)	Find the coordinates of E, if C is the mid-point of BE.
d)	Prove that $CB \perp AB$.
e)	Determine what angle a line drawn through BA makes with the positive x -axis.
4	The points $A(-4;0)$, $B(0;2)$, $C(1;0)$ and $D(0;-2)$ form the quadrilateral shown below.

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a)	Prove that $\triangle ABC$ is right-angled.
b)	What kind of quadrilateral is ABCD? Use the lengths of the sides to justify your answer.
5	A(0; 4), B(3; 1), C(-3; -5) and D(-6; -2) are the vertices of a quadrilateral in a Cartesian plane.
a)	Prove that ABCD is a rectangle. (Show ALL the calculations.)
b)	Hence determine the coordinates of the point of intersection of the diagonals of rectangle ABCD.
7	ABCD is a quadrilateral with vertices A(-1; 4), B(-2; -1), C(3; -2) and D(4; 3). M is the midpoint of AC.

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a)	Show that $AB \parallel CD$ and $AD \parallel BC$.
b)	Show that the midpoint of AC and BD coincide (i.e. they are the same points).
c)	Show that $\angle ABC$ is a right angle.
d)	Calculate the area of $\triangle ABC$
e)	Prove that ABCD is a square.
f)	Determine the equation of AB.
g)	Find the x-intercept of AB.
h)	Find the angle of inclination of AB (i.e. the size of θ) correct to one decimal place.